

The Degrees of Freedom of the Interference Channel with a Cognitive Relay under Delayed Feedback

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Abstract

This paper studies the interference channel with a cognitive relay (ICCR) under delayed feedback. Three types of delayed feedback are studied: delayed channel state information at the transmitter (CSIT), delayed output feedback, and delayed Shannon feedback. Outer bounds are derived for the DoF region of the two-user multiple-input multiple-output (MIMO) ICCR with delayed feedback as well as without feedback. For the single-input single-output (SISO) scenario, optimal schemes are proposed based on retrospective interference alignment. It is shown that while a cognitive relay *without* feedback cannot extend the sum-DoF beyond 1 in the two-user SISO interference channel, delayed feedback in the same scenario can extend the sum-DoF to $4/3$. For the MIMO case, achievable schemes are obtained via extensions of retrospective interference alignment, leading to DoF regions that meet the respective upper bounds.

I. INTRODUCTION

Cognitive radio is a subject of intense interest motivated by its potential for better usage of spectral resources. To explore the fundamental limits of such channels, and to make use of powerful techniques

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developed for capacity of channels with state known at transmitter, some information-theoretic cognitive models allow a cognitive node to possess non-causal knowledge of data originating elsewhere. Interestingly, in the recent years applications have emerged where knowledge of another nodes' data prior to transmission is indeed practically viable. Examples include heterogeneous networks or coordinated networks, where some base stations can possess knowledge of the messages of other base stations by coordination. Other examples involve layered cell structures, where macro base stations can know the messages of pico base stations that are routed from the macro base station over backhaul links. Such heterogeneous or coordinated networks can be modeled by interference channels with cognitive transmitters [1].

Contrary to the model in [1], when a cognitive transmitter does not have its intended receiver, it is called cognitive relay and helps other transmitters in a way of reducing the effective interference at the receivers. In this paper, we consider the interference channel with a cognitive relay (ICCR)¹ where transmitter-receiver pairs constitute an interference channel and the cognitive relay helps the transmitters.

Previous works in this area have generally focused on perfect and instantaneous channel state information at transmitter (CSIT). However, feedback delays are often present in real systems and make feedback information outdated. Fortunately, the usefulness of delayed CSIT has been explored in various channel models [15]–[25]. The ICCR with delayed feedback, nevertheless, has not received much attention despite its importance.

A. Past Work

The ICCR was first considered in [2] where an achievable rate region via a combination of dirty paper coding [3] and beamforming was reported. In [4], a new achievable region was presented by a combination of the Han-Kobayashi coding scheme [5] and dirty paper coding, and an outer bound for the Gaussian ICCR was derived. For a discrete memoryless (DM) ICCR, an outer bound was first derived in

¹It is also known by the name cognitive relay-assisted interference channel.

[6] and then improved achievable rate regions and outer bounds were reported in [7]–[10]. The capacity region of DM-ICCR is known in very strong and strong interference regime [7], [8], but it still remains unknown under general channel conditions.

When capacity remains intractable, the degrees of freedom (DoF) are often used to understand the asymptotic characteristics of the capacity. The DoF is defined as the ratio of the capacity of the channel of interest to a simple SISO Gaussian channel capacity, when transmit power goes infinity. The DoF of ICCR has been studied in [4], [11], [12]. It was proved in [4] that the two-user Gaussian ICCR has DoF 2 almost surely if perfect and instantaneous CSIT and CSI at receiver (CSIR) are available, which implies that each receiver does not suffer from interference in an asymptotic sense. For the K users with perfect CSIT and CSIR, achievable sum DoF and outer bounds of the sum DoF were derived in [11], [12]. Although a conventional relay cannot increase the DoF [13], the authors of [12] showed that a cognitive relay can improve DoF unlike a conventional relay; the optimal sum DoF for K users with perfect CSIT is $\frac{K+1}{2}$ if K is odd while the sum DoF for the K -user interference channel with perfect CSIT is $\frac{K}{2}$ by interference alignment [14].

The usefulness of delayed CSIT has been first demonstrated in [15] for multiple-input single-output broadcast channel (BC). In [15], the base station exploits the delayed CSI to estimate the interference at each receiver in the previous transmission (i.e., the side information at the receivers) and then retrospectively align the interfering signals with the help of the side information. For multiple-input multiple-output (MIMO) BC with delayed CSIT, an outer bound of the DoF region with K users and the DoF region with two users were derived in [16], and sum DoF for a three-user case was obtained in [17]. New retrospective interference alignment schemes for an interference channel and an X channel with delayed CSIT and delayed output feedback were proposed in [18], and the sum DoF was derived. The achievable DoF reported in [18] was improved in [19], [20]. Recently, [21] proved the usefulness of ergodic interference alignment in a K -user interference channel when only delayed feedback is available and showed that the sum DoF of 2 can be achievable as K goes to infinity, which is the best DoF result until now in a K -user

interference channel with delayed CSIT. derived in [22]–[24]. In [22], the DoF region with delayed CSIT was derived for general MIMO interference channel with an arbitrary numbers of antennas. The authors of [22] showed that Shannon feedback, which has both output feedback and delayed CSIT, strictly enlarges the DoF region of the MIMO interference channel compared to the case with delayed CSIT only [23]. For delayed local CSIT, an achievable DoF region of MIMO interference channel was derived in [24]. In [25], the authors presented a hybrid CSIT model where one transmitter has perfect and instantaneous knowledge of channel matrices corresponding to one user while the other transmitter has only delayed CSI corresponding to the other user, and derived the DoF region of the MIMO interference channel with hybrid CSIT. Moreover, the DoF regions of MIMO interference channel and broadcast channel without CSIT were derived in [26], and in addition to MIMO interference channel and broadcast channel, the DoF region of a cognitive radio channel without CSIT was reported in [27].

B. Main contribution

In this paper, we consider the interference channel with cognitive relay in the presence of various types of delayed feedback at the transmitter in independent and identically distributed (i.i.d.) fading channels. In all cases perfect CSIR is assumed. Unless explicitly mentioned otherwise, a two-user system is considered. The types of delayed feedback information (including no feedback) are

- Delayed CSIT: Transmitters and a cognitive relay know all channels after one sample delay.
- Delayed output feedback: Transmitters and a cognitive relay know output of their intended receiver after one sample delay.
- Delayed Shannon feedback: Transmitters and a cognitive relay know all channel gains and the output of intended receiver after one sample delay.
- No feedback: Both transmitters and a cognitive relay do not have any channel information.

For each type of delayed feedback, an outer bound for the DoF region is derived. Focusing on the special case of the single-input single-output (SISO), a scheme is proposed that achieves the outer bound

based on the retrospective interference alignment for each type of feedback. From the derived DoF region, it is shown that the sum DoF in the single-antenna network is $\frac{4}{3}$ for delayed CSIT, delayed output feedback, and delayed Shannon feedback with the help of a cognitive relay, compared with the sum-DoF of the interference channel which is only 1 regardless of availability of CSIT. It is also shown that a cognitive relay does not extend the DoF region in the absence of CSIT.

The proposed retrospective interference alignment scheme is extended to the MIMO case. It is shown that for the three types of delayed feedback information, the DoF regions achieved by the proposed retrospective interference alignment scheme are similar, matching the DoF outer bound for all antenna configurations. Comparing with the DoF region when delayed feedback information is not available at both the transmitters and the cognitive relay, the delayed feedback information is useful for expanding the DoF region when $M_r < M_t + M_c$ where M_t , M_c , and M_r are the numbers of antennas at the transmitter, the relay, and the receiver, respectively. If delayed feedback is not available at the cognitive relay, the optimal DoF region is shown to be achievable except when $M_t < M_r < M_t + M_c$ by the proposed retrospective interference alignment scheme. Our results quantitatively reveal the DoF gain from the cognitive relay according to antenna configurations when only delayed feedback information is available.

Finally, we compare the sum DoF of ICCR with those of broadcast channel and interference channel when only delayed CSIT is available. With the help of the cognitive relay, ICCR has an enlarged DoF region compared to MIMO interference channel. Furthermore, as a corollary of the above-mentioned results, lower and upper bounds are derived for the sum DoF of a cognitive interference channel which is also known as a interference channel with a cognitive transmitter.

C. Paper Organization

This paper is organized as follows. Section II describes the system model with various types of delayed feedback information. In Section III, focusing on the SISO model as a special case, we propose a modified retrospective interference alignment scheme achieving the outer bound for SISO model under various types of delayed feedback information. Section IV derives the DoF regions for the multiple antenna scenarios.

Section V derives the achievable DoF region when delayed feedback information is not available at the cognitive relay. In Section VI, we derive the DoF outer bounds with and without delayed feedback information. Section VII discusses a comparison with BC and IC under delayed CSIT and an extension to cognitive interference channel. Section VIII concludes the paper.

II. SYSTEM MODEL

This paper considers a MIMO network consisting of two transmitters with M_t antennas, two receivers with M_r antennas, and a cognitive relay with M_c antennas as shown in Fig. 1, where the desired and interference links are represented by solid and dashed lines, respectively. The links experience i.i.d. Rayleigh fading. Transmitter a has message W_a intended for Receiver a , Transmitter b has message W_b intended for Receiver b , and the cognitive relay has both W_a and W_b non-causally, where the messages W_a and W_b are independent. Channel outputs at time slot t are

$$Y_{a,t} = \mathbf{H}_{aa,t}X_{a,t} + \mathbf{H}_{ab,t}X_{b,t} + \mathbf{H}_{ac,t}X_{c,t} + Z_{a,t}, \quad (1a)$$

$$Y_{b,t} = \mathbf{H}_{ba,t}X_{a,t} + \mathbf{H}_{bb,t}X_{b,t} + \mathbf{H}_{bc,t}X_{c,t} + Z_{b,t}, \quad (1b)$$

where $Y_{j,t} = [Y_{j[1],t}, \dots, Y_{j[M_r],t}]^T \in \mathbb{C}^{M_r \times 1}$, $j \in \{a, b\}$, is the received signal at Receiver j , $Y_{j[\ell],t}$ is the ℓ -th element of $Y_{j,t}$, $X_{i,t} \in \mathbb{C}^{M_t \times 1}$, $i \in \{a, b\}$, is the transmitted signal from Transmitter i , $X_{c,t} \in \mathbb{C}^{M_c \times 1}$ is the transmitted signal from the cognitive relay, $\mathbf{H}_{ji,t} \in \mathbb{C}^{M_r \times M_t}$ is the time varying channel matrix from Transmitter i to Receiver j , $\mathbf{H}_{jc,t} \in \mathbb{C}^{M_r \times M_c}$ is time varying channel matrix from the cognitive relay to Receiver j , and $Z_{j,t}$ ² is an i.i.d. circular symmetric complex Gaussian noise, $\mathcal{CN}(0, I_{M_r})$, at Receiver j . We assume that all channel coefficients are i.i.d. circular symmetric complex Gaussian random variables with zero mean and unit variance, $\mathcal{CN}(0, 1)$.

In this paper, we assume perfect CSIR. Certain feedback information is available at the transmitters and cognitive relay with delay, represented by the following four cases, where $i \in \{a, b\}$ and t is the time index:

²This noise terms can be ignored since this paper considers a high signal-to-noise (SNR) model.

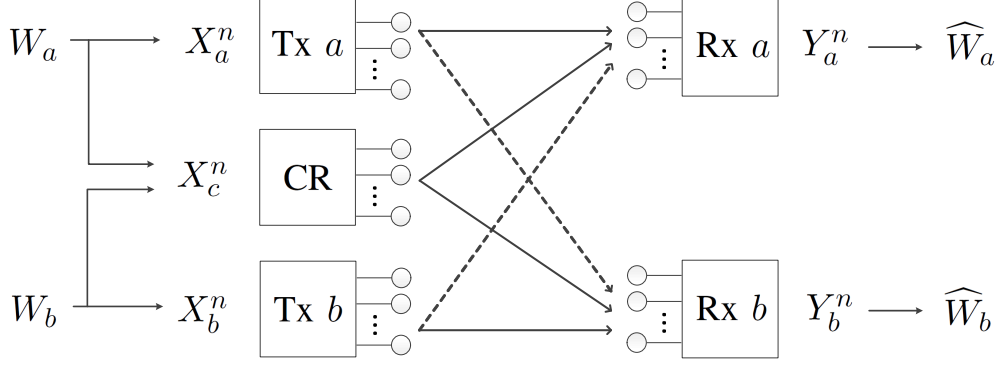


Fig. 1. A MIMO interference channel with a cognitive relay.

- 1) Delayed CSIT: $X_{i,t} = f_{i,t}(W_i, \mathcal{H}^{t-1})$, $X_{c,t} = f_{c,t}(W_a, W_b, \mathcal{H}^{t-1})$
- 2) Delayed output feedback: $X_{i,t} = f_{i,t}(W_i, Y_i^{t-1})$, $X_{c,t} = f_{c,t}(W_a, W_b, Y_a^{t-1}, Y_b^{t-1})$
- 3) Delayed Shannon feedback: $X_{i,t} = f_{i,t}(W_i, Y_i^{t-1}, \mathcal{H}^{t-1})$, $X_{c,t} = f_{c,t}(W_a, W_b, Y_a^{t-1}, Y_b^{t-1}, \mathcal{H}^{t-1})$
- 4) No feedback: $X_{i,t} = f_{i,t}(W_i)$, $X_{c,t} = f_{c,t}(W_a, W_b)$.

Each message $W_i \in \{1, 2, \dots, 2^{nR_i(P)}\}$ is uniformly distributed, $f_{i,t}$ and $f_{c,t}$ are, respectively, encoding functions at Transmitter i and the cognitive relay for channel use t and \mathcal{H}_t is the set of all channel matrices at time index t , i.e.,

$$\mathcal{H}_t \triangleq \{\mathbf{H}_{aa,t}, \mathbf{H}_{ab,t}, \mathbf{H}_{ac,t}, \mathbf{H}_{ba,t}, \mathbf{H}_{bb,t}, \mathbf{H}_{bc,t}\},$$

$$\mathcal{H}^t \triangleq \{\mathcal{H}_1, \mathcal{H}_2, \dots, \mathcal{H}_t\}.$$

$X_{i,t}$ and $X_{c,t}$ should satisfy the power constraint $\mathbb{E}[||X_{i,t}||^2] \leq P$ and $\mathbb{E}[||X_{c,t}||^2] \leq P$, respectively where $i \in \{a, b\}$.

Receiver i decodes the message from the received signal with a decoding function g_i such that $\widehat{W}_i = g_i(Y_i^n, \mathcal{H}^n)$.

A rate pair $(R_a(P), R_b(P))$ is achievable if there exists a sequence of codes $(2^{nR_a(P)}, 2^{nR_b(P)}, n)$ whose average probability of error goes to zero as $n \rightarrow \infty$. The capacity region $\mathcal{C}(P)$ is defined as the set of all achievable rate pairs $(R_a(P), R_b(P))$, and the DoF region can be defined from the capacity

TABLE I
DoF NOTATIONS FOR INTERFERENCE CHANNEL WITH COGNITIVE RELAY

\mathcal{D}_{no}	DoF region with no feedback
\mathcal{D}_{CSI}	DoF region with delayed CSIT
$\mathcal{D}_{\text{output}}$	DoF region with delayed output feedback
$\mathcal{D}_{\text{Shannon}}$	DoF region with delayed Shannon feedback
$\mathcal{D}_{\text{perfect}}$	DoF region with perfect CSIT
$\mathcal{D}'_{\text{delay}\backslash\text{CR}}$	Achievable DoF region with delayed feedback unavailable at CR
$\bar{\mathcal{D}}_{\text{delay}}$	DoF outer bound with delayed feedback
$\bar{\mathcal{D}}_{\text{no}}$	DoF outer bound with no feedback

region as

$$\mathcal{D} = \left\{ (d_a, d_b) \in \mathbb{R}_+^2 \mid \forall (w_a, w_b) \in \mathbb{R}_+^2, \right. \\ \left. w_a d_a + w_b d_b \leq \limsup_{P \rightarrow \infty} \frac{1}{\log_2 P} \left[\sup_{(R_a(P), R_b(P)) \in \mathcal{C}(P)} w_a R_a(P) + w_b R_b(P) \right] \right\}.$$

The element ℓ of the received vector $Y_{i,t}$ at time index t is denoted as $Y_{i[\ell],t}$. In a similar manner, a subset of elements from this vector is denoted as follows:

$$Y_{i[\ell_1:\ell_2],t} \triangleq \{Y_{i[\ell_1],t}, Y_{i[\ell_1+1],t}, \dots, Y_{i[\ell_2],t}\}$$

In the same manner, we define a sequence of vectors over all (causal) time that select only a subset of the antennas:

$$Y_{i[\ell_1:\ell_2]}^t \triangleq \{Y_{i[\ell_1]}^t, Y_{i[\ell_1+1]}^t, \dots, Y_{i[\ell_2]}^t\}$$

In the special case where only one antenna is selected across time we have $Y_{i[\ell]}^t = \{Y_{i[\ell],1}, Y_{i[\ell],2}, \dots, Y_{i[\ell],t}\}$.

$g(x) = o(f(x))$ denotes that functions $g(\cdot), f(\cdot)$ have the following tail characteristic: $\lim_{x \rightarrow \infty} \frac{g(x)}{f(x)} = 0$. Several specialized notations are shown in Table I that distinguish the DoF regions under various conditions.

III. SISO DoF WITH DELAYED FEEDBACK

This section focuses on the SISO special case, i.e., $M_t = M_r = M_c = 1$. We propose a modified retrospective interference alignment scheme achieving the DoF outer bound of the Gaussian i.i.d. fading SISO interference channel with a cognitive relay. This is done on the one hand when any of the three kinds of feedback information is available, and on the other hand when no feedback is available. Each receiver is assumed to have perfect CSI.

A. Delayed CSIT

We now assume the transmitters and cognitive relay have perfect knowledge of all channel gains after one time slot delay.

Theorem 1: The DoF region of the SISO ICCR with delayed CSIT \mathcal{D}_{CSI} is

$$\mathcal{D}_{\text{CSI}} = \left\{ (d_a, d_b) \in \mathbb{R}_+^2 : d_a + \frac{d_b}{2} \leq 1, \frac{d_a}{2} + d_b \leq 1 \right\} \quad (2)$$

where $M_t = M_r = M_c = 1$.

Proof: The outer bound of DoF region given in (2) will be derived in Theorem 6 of Section VI. We propose a coding scheme that achieves a $(d_a, d_b) = (\frac{2}{3}, \frac{2}{3})$ DoF pair almost surely. The coding scheme is a modified version of the retrospective interference alignment in [15] for our channel model. The DoF tuple achieved by this scheme is a point on the DoF region as shown in Fig. 2. Then, we can also achieve the entire DoF region using time sharing.

Now, we show that the $(\frac{2}{3}, \frac{2}{3})$ DoF pair is achievable under delayed CSIT. Time slots are partitioned into groups of three, and each transmitter sends two symbols during the 3 time slots, thus DoF of 2/3 is achieved per user. The transmit symbols of Transmitter a are denoted as S_{1a} and S_{2a} , and the transmit symbols for Transmitter b are Q_{1b} and Q_{2b} . The transmission mechanism is as follows: in the first time slot Transmitter a and the cognitive relay transmit (different) random linear combinations of S_{1a}, S_{2a} , while Transmitter b is silent. Neglecting the noise terms, the received signals are:

$$Y_{a,1} = H_{aa,1}(u_{1a,1}S_{1a} + u_{2a,1}S_{2a}) + H_{ac,1}(v_{1c,1}S_{1a} + v_{2c,1}S_{2a}), \quad (3a)$$

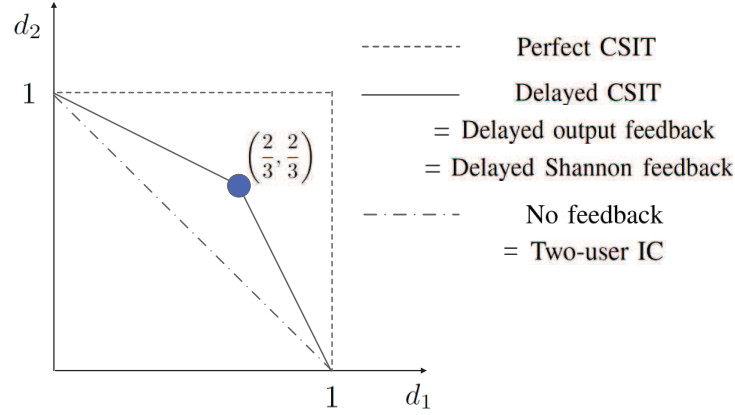


Fig. 2. The DoF region of the SISO Gaussian interference channel with and without a cognitive relay.

$$Y_{b,1} = H_{ba,1}(u_{1a,1}S_{1a} + u_{2a,1}S_{2a}) + H_{bc,1}(v_{1c,1}S_{1a} + v_{2c,1}S_{2a}). \quad (3b)$$

All precoding variables are chosen so that power constraints are satisfied, but so that $\frac{u_{1a,1}}{u_{2a,1}} \neq \frac{v_{1c,1}}{v_{2c,1}}$. In time slot 2, a similar action takes place, except Transmitter b and the cognitive relay transmit and Transmitter a is silent.

$$Y_{a,2} = H_{ab,2}(u_{1b,2}Q_{1b} + u_{2b,2}Q_{2b}) + H_{ac,2}(v_{1c,2}Q_{1b} + v_{2c,2}Q_{2b}), \quad (4a)$$

$$Y_{b,2} = H_{bb,2}(u_{1b,2}Q_{1b} + u_{2b,2}Q_{2b}) + H_{bc,2}(v_{1c,2}Q_{1b} + v_{2c,2}Q_{2b}). \quad (4b)$$

where similar conditions on the precoding variables are imposed. Finally, in time slot 3, Transmitter a and Transmitter b transmit but the relay is silent. Using delayed CSIT, the transmitters respectively transmit the received signal at their non-intended receiver during the initial transmission, appropriately scaled to account for power constraints.

$$Y_{a,3} = H_{aa,3}(p_1 Y_{b,1}) + H_{ab,3}(p_2 Y_{a,2}), \quad (5a)$$

$$Y_{b,3} = H_{ba,3}(p_1 Y_{b,1}) + H_{bb,3}(p_2 Y_{a,2}). \quad (5b)$$

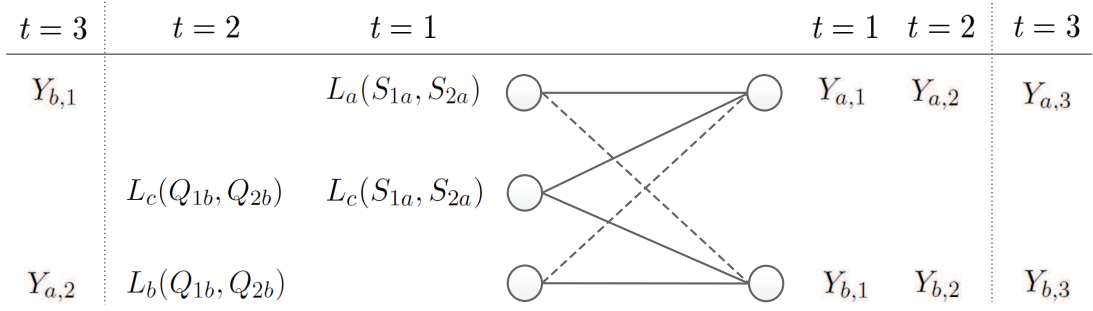


Fig. 3. Achievable scheme for the delayed CSIT case, $L(x, y)$ is a random linear combination of x and y .

Subtracting $H_{ab,3}(p_2 Y_{a,2})$ from $Y_{a,3}$ with the received signal at time index $t = 2$, Receiver a can obtain the interference-free signal $Y_{b,1}$ as

$$Y_{b,1} = \frac{Y_{a,3} - H_{ab,3}(p_2 Y_{a,2})}{H_{aa,3} p_1}.$$

The signaling scheme is depicted in Fig. 3. We can readily know that $Y_{a,1}$ and $Y_{b,1}$ are almost surely linearly independent since channel gains are independently drawn from the same continuous distribution and $u_{ja,1}$ and $v_{ja,1}, j \in \{1, 2\}$, are also random and independent. Thus, Receiver a has two independent equations given by linear combinations of two variables S_{1a} and S_{2a} so that it can decode two symbols. Similarly, since Receiver b can obtain $Y_{a,2}$, Receiver b also has two independent equations $Y_{a,2}$ and $Y_{b,2}$ of two variables intended for Receiver b , and hence Receiver b can decode two symbols Q_{1b} and Q_{2b} . Consequently, at the end of transmission, each receiver can achieve $\frac{2}{3}$ DoF (i.e., two symbols over 3 time slots) almost surely. In other words, the sum DoF is $\frac{4}{3}$. ■

Remark 1: The DoF region under perfect CSIT at the transmitters and cognitive relay is [4]

$$\mathcal{D}_{\text{perfect}} = \left\{ (d_a, d_b) \in \mathbb{R}_+^2 : d_a \leq 1, d_b \leq 1 \right\} \quad (6)$$

which is shown in Fig. 2 as a reference. With perfect instantaneous CSI at the transmitters and cognitive relay, sum DoF is 2 almost surely, which is as if receivers are free from interference. The DoF achieving strategy is interference pre-cancellation via the relay's non-causal knowledge of the messages. On the

other hand, the Gaussian SISO interference channel without cognitive relay has sum DoF of 1 regardless of whether transmitters have CSI.

Remark 2: Theorem 1 indicates that a cognitive relay can increase DoF even with delayed CSIT although the amount of DoF increased by a cognitive relay is limited compared to the case of perfect CSIT; the SISO ICCR with delayed CSIT has total $\frac{4}{3}$ DoF at most.

B. Delayed Output Feedback

Each receiver feeds its output back to its transmitter so that each transmitter knows only the output of the intended receiver after one time slot delay. The cognitive relay also has the output feedback from both the receivers after one time slot delay.

Theorem 2: The DoF region of the SISO ICCR with delayed output feedback is

$$\mathcal{D}_{\text{output}} = \mathcal{D}_{\text{CSI}}. \quad (7)$$

where $M_t = M_r = M_c = 1$.

Proof: The outer bound is determined by $d_a + \frac{d_b}{2} \leq 1$, $\frac{d_a}{2} + d_b \leq 1$ that will be derived in Theorem 6 of Section VI. We propose a scheme that achieves $(d_a, d_b) = (\frac{2}{3}, \frac{2}{3})$ DoF pair almost surely, a point that is on the outer bound, and then all other points on the outer bound are achieved via time sharing.

Similar to the delayed CSIT case, the achievable scheme needs 3 time slots. At time slot 1 and 2, the signaling follows Section III-A. In time slot 3, however, a different signaling is used where the transmitter utilizes the output feedback from the receiver instead of constructing a linear combination of previous symbols based on delayed CSI. Each transmitter transmits the output fed back from the intended receiver, appropriately scaled to satisfy the power constraints.

$$Y_{a,3} = H_{aa,3}(p_1 Y_{a,2}) + H_{ab,3}(p_2 Y_{b,1}), \quad (8a)$$

$$Y_{b,3} = H_{ba,3}(p_1 Y_{a,2}) + H_{bb,3}(p_2 Y_{b,1}), \quad (8b)$$

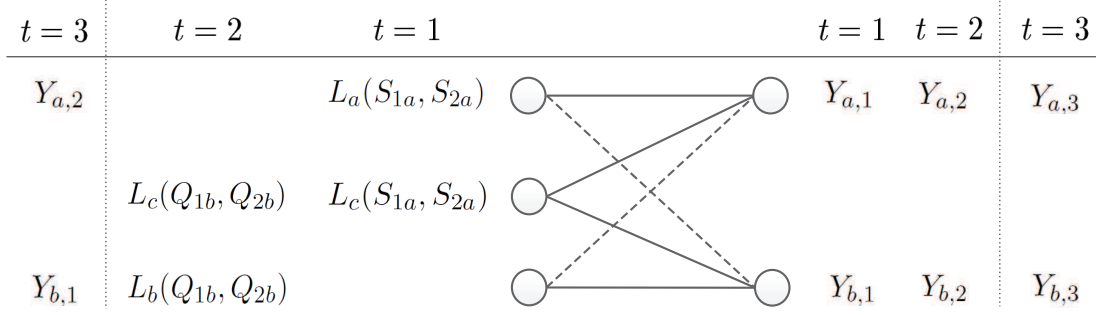


Fig. 4. Achievable scheme for the delayed output feedback case, $L(x, y)$ is a random linear combination of x and y .

Subtracting $H_{aa,3}(p_1 Y_{a,2})$ from $Y_{a,3}$ with the received signal at time index $t = 2$, Receiver a can obtain the interference-free signal $Y_{b,1}$ as

$$Y_{b,1} = \frac{Y_{a,3} - H_{aa,3}(p_1 Y_{a,2})}{H_{ab,3} p_2}.$$

The signaling scheme is shown in Fig. 4. Because Receiver a almost surely has two linearly independent equations $Y_{a,1}$ and $Y_{b,1}$ that are linear combinations of two symbols S_{1a} and S_{2a} , Receiver a is able to decode the two symbols. Similarly, because Receiver b can obtain $Y_{a,2}$ and almost surely has two linearly independent equations $Y_{a,2}$ and $Y_{b,2}$ of two symbols, Receiver b can decode two symbols Q_{1b} and Q_{2b} . As a result, each receiver can achieve $\frac{2}{3}$ DoF almost surely, and we can achieve $\frac{4}{3}$ sum DoF. ■

C. Delayed Shannon Feedback

Shannon feedback refers to a strictly causal feedback that gives each transmitter all the channel state information as well as the received value at the intended receiver. The cognitive relay has also the delayed Shannon feedback from the receivers after one time slot delay.

Theorem 3: The DoF region of the SISO ICCR with delayed Shannon feedback is

$$\mathcal{D}_{\text{Shannon}} = \mathcal{D}_{\text{CSI}}. \quad (9)$$

Proof: The outer bound for delayed Shannon feedback is the same as that for delayed CSIT feedback or delayed output feedback. The outer bound is presented in Theorem 6 of Section VI. The outer bound can

be achieved for the cases of delayed CSIT and delayed output feedback from Theorem 1 and Theorem 2. Therefore, the outer bound is also achievable because we can use both the delayed CSIT and the output feedback information. ■

Remark 3: For the SISO case, the proposed scheme does not entail any delayed feedback information at the cognitive relay. Therefore, the optimal DoF region of the SISO ICCR can be obtained even if the cognitive relay does not have delayed feedback.

D. No Feedback

Corollary 1: The DoF region for the SISO ICCR with no feedback is

$$\mathcal{D}_{\text{no}} = \left\{ (d_a, d_b) \in \mathbb{R}_+^2 : d_a + d_b \leq 1 \right\}. \quad (10)$$

Proof: The DoF outer bound is $d_a + d_b \leq 1$ that will be proved in Corollary 5 of Section VI. The outer bound is achievable via time division multiplexing (TDM) when the transmitters and cognitive relay do not have any feedback information. ■

The result is true irrespective of the number of transmit or receive antennas. In section IV-D, we will show that TDM is also DoF optimal for the MIMO case.

Remark 4: The DoF region in Corollary 1 is the same as that of the SISO interference channel without a cognitive relay. This shows the cognitive relay in the SISO case has no effect on DoF in the absence of CSIT.

IV. MIMO DoF WITH DELAYED FEEDBACK

This section extends the modified retrospective interference alignment scheme and applies it to multi-antenna nodes. We derive achievable DoF region for four types of feedback information (including no feedback). Each transmitter has M_t antennas, each receiver has M_r antennas, and the cognitive relay has M_c antennas. We continue to assume perfect CSIR.

TABLE II
THE FIVE CONDITIONS ACCORDING TO ANTENNA CONFIGURATIONS

$M_t + M_c \leq M_r$	$\frac{M_t + M_c}{2} \leq M_r < M_t + M_c$		$M_r < \frac{M_t + M_c}{2}$	
	$M_r > M_t$	$M_r \leq M_t$	$M_r > M_t$	$M_r \leq M_t$
Condition I	Condition II	Condition III	Condition IV	Condition V

A. Delayed CSIT

The transmitters and cognitive relay have perfect knowledge of all channel information after one time slot delay. The analysis is divided into five categories according to antenna configuration (see Table II).

Theorem 4: The DoF region of the MIMO ICCR with delayed CSIT is

$$\mathcal{D}_{\text{CSI}} = \left\{ (d_a, d_b) \in \mathbb{R}_+^2 : d_a \leq \min(M_r, M_t + M_c), d_b \leq \min(M_r, M_t + M_c), \right. \\ \left. \frac{d_a}{\min(M_r, M_t + M_c)} + \frac{d_b}{\min(2M_r, M_t + M_c)} \leq \frac{\min(M_r, 2M_t + M_c)}{\min(M_r, M_t + M_c)}, \right. \\ \left. \frac{d_a}{\min(2M_r, M_t + M_c)} + \frac{d_b}{\min(M_r, M_t + M_c)} \leq \frac{\min(M_r, 2M_t + M_c)}{\min(M_r, M_t + M_c)} \right\}$$

where M_t , M_r , and M_c are the number of antennas at the transmitter, the receiver and the cognitive relay, respectively.

Proof: We show the achievable DoF region according to the classified conditions, and compare the achievable DoF region with the DoF outer bound that will be derived in Theorem 6 of Section VI. The delayed CSIT is not used in the achievable scheme for Condition I, but we exploit delayed CSIT for Condition II, III, IV and V.

1) Condition I: $M_t + M_c \leq M_r$

In this case, the DoF outer bound with delayed CSIT is constructed from $d_a \leq M_t + M_c$, $d_b \leq M_t + M_c$, and $d_a + d_b \leq \min(M_r, 2M_t + M_c)$. Using a similar approach of decomposing an interference channel into multiple access channel (MAC) [28], we decompose the ICCR into MACs.

The two users and the cognitive relay each use their own codewords. Since each receiver can decode the maximum of $\min(M_r, 2M_t + M_c)$ signals, the two transmitters and cognitive relay send total $\min(M_r, 2M_t + M_c)$ messages, then each receiver can decode all signals. Consequently, the optimal DoF region is obtained and the total sum DoF becomes $\min(M_r, 2M_t + M_c)$.

2) Condition II: $\frac{M_t + M_c}{2} \leq M_r < M_t + M_c$ and $M_r > M_t$

A DoF outer bound with delayed CSIT for this case is given by $\frac{d_a}{M_t + M_c} + \frac{d_b}{M_r} \leq 1$ and $\frac{d_a}{M_r} + \frac{d_b}{M_t + M_c} \leq 1$.

1. We can show that the DoF pair $\left(\frac{(M_t + M_c)M_r}{M_r + M_t + M_c}, \frac{(M_t + M_c)M_r}{M_r + M_t + M_c}\right)$ is achievable, which lies on the intersection of $\frac{d_a}{M_t + M_c} + \frac{d_b}{M_r} \leq 1$ and $\frac{d_a}{M_r} + \frac{d_b}{M_t + M_c} \leq 1$ on the DoF outer bound. First, for each time slot $t \in \{1, \dots, M_r\}$, Transmitter a sends random linear combinations of $M_t + M_c$ independent symbols, and the cognitive relay sends different random linear combinations of the $M_t + M_c$ independent symbols. The received signals at time index $t \in \{1, \dots, M_r\}$ can be represented as

$$Y_{a,t} = \mathbf{H}_{aa,t} \mathbf{U}_{a,t} S_{a,t} + \mathbf{H}_{1c,t} \mathbf{V}_{c,t} S_{a,t}, \quad (11a)$$

$$Y_{b,t} = \mathbf{H}_{ba,t} \mathbf{U}_{a,t} S_{a,t} + \mathbf{H}_{2c,t} \mathbf{V}_{c,t} S_{a,t}, \quad (11b)$$

where $\mathbf{U}_{a,t}$ is a randomly chosen $M_t \times (M_t + M_c)$ matrix with rank M_t , $\mathbf{V}_{c,t}$ is a randomly chosen $M_c \times (M_t + M_c)$ matrix with rank M_c , $S_{a,t}$ is an $(M_t + M_c) \times 1$ symbol vector for Receiver a at time index t , the transmissions are appropriately scaled to satisfy the power constraint, and noise terms are omitted since noise does not affect DoF. Because Receiver a obtains M_r linear combinations of the desired $M_t + M_c$ variables at each time index, Receiver a has total M_r^2 independent linear equations of the $(M_t + M_c)M_r$ desired symbols during M_r time slots.

Then in each time slot $t \in \{M_r + 1, \dots, 2M_r\}$, Transmitter b and the cognitive relay send different random linear combinations of $M_t + M_c$ symbols intended for Receiver b . The received signals at time index $t \in \{M_r + 1, \dots, 2M_r\}$ are

$$Y_{a,t} = \mathbf{H}_{ab,t} \mathbf{U}_{b,t} Q_{b,t} + \mathbf{H}_{1c,t} \mathbf{V}_{c,t} Q_{b,t}, \quad (12a)$$

$$Y_{b,t} = \mathbf{H}_{bb,t} \mathbf{U}_{b,t} Q_{b,t} + \mathbf{H}_{2c,t} \mathbf{V}_{c,t} Q_{b,t}, \quad (12b)$$

where $\mathbf{U}_{b,t}$ is a randomly chosen $M_t \times (M_t + M_c)$ matrix with rank M_t , $\mathbf{V}_{c,t}$ is a randomly chosen $M_c \times (M_t + M_c)$ matrix with rank M_c , $Q_{b,t}$ is an $(M_t + M_c) \times 1$ symbol vector for Receiver b at time index t , all coefficients are appropriately selected to satisfy the power constraint, and noise terms are omitted. Receiver b obtains total M_r^2 independent linear combinations of the $(M_t + M_c)M_r$ desired variables during M_r time slots.

At time index $t \in \{2M_r + 1, \dots, M_t + M_c + M_r\}$, Transmitter a , Transmitter b , and the cognitive relay transmit $X_{a,t} = [Y_{b[t-2M_r],1}, \dots, Y_{b[t-2M_r],M_t}]^T$, $X_{b,t} = [Y_{a[t-2M_r],M_r+1}, \dots, Y_{a[t-2M_r],M_r+M_t}]^T$ and $X_{c,t} = [Y_{b[t-2M_r],M_t+1} + Y_{a[t-2M_r],M_r+M_t+1}, \dots, Y_{b[t-2M_r],M_r} + Y_{a[t-2M_r],2M_r}, 0, \dots, 0]^T$, respectively, using delayed CSI. Note that the cognitive relay transmits $X_{c,t}$ using only $M_r - M_t$ antennas. The transmissions are appropriately scaled to satisfy the power constraint. The received signals at $t \in \{2M_r + 1, \dots, M_t + M_c + M_r\}$ are

$$Y_{a,t} = \mathbf{H}_{aa,t}X_{a,t} + \mathbf{H}_{ab,t}X_{b,t} + \mathbf{H}_{ac,t}X_{c,t}, \quad (13a)$$

$$Y_{b,t} = \mathbf{H}_{ba,t}X_{a,t} + \mathbf{H}_{bb,t}X_{b,t} + \mathbf{H}_{bc,t}X_{c,t}, \quad (13b)$$

where noise terms are omitted. Since the interfering terms are comprised of the past received signal in previous slots, each receiver can eliminate the interfering terms using the received signals in the previous time slots and obtain $(M_t + M_c - M_r)M_r$ linearly independent interference-free signals during $M_t + M_c - M_r$ time slots. Therefore, each receiver has $(M_t + M_c)M_r$ linearly independent equations involving $(M_t + M_c)M_r$ symbols and thus we can obtain the DoF pair $\left(\frac{(M_t+M_c)M_r}{M_t+M_c+M_r}, \frac{(M_t+M_c)M_r}{M_t+M_c+M_r}\right)$ which is the same achievable DoF pair in Condition II. The other points on the DoF outer bound can be also achieved via time sharing.

- 3) Condition III: $\frac{M_t+M_c}{2} \leq M_r < M_t + M_c$ and $M_r \leq M_t$

We show that the $\left(\frac{(M_t+M_c)M_r}{M_t+M_c+M_r}, \frac{(M_t+M_c)M_r}{M_t+M_c+M_r}\right)$ DoF pair is achievable, which is an intersection point of $\frac{d_a}{M_t+M_c} + \frac{d_b}{M_r} \leq 1$ and $\frac{d_a}{M_r} + \frac{d_b}{M_t+M_c} \leq 1$ on the DoF outer bound.

First, we spend $2M_r$ time slots. At each time index $t \in \{1, \dots, M_r\}$, Transmitter a sends random

linear combinations of $M_t + M_c$ independent symbols, and the cognitive relay sends different random linear combinations of the $M_t + M_c$ independent symbols. Since Receiver a obtains M_r linear combinations of the desired $M_t + M_c$ variables at each time index, Receiver a has total M_r^2 independent linear equations of the $(M_t + M_c)M_r$ desired symbols during M_r time slots. At each time index $t \in \{N + 1, \dots, 2M_r\}$, Transmitter b and the cognitive relay send different random linear combinations of $M_t + M_c$ symbols intended for Receiver b . Receiver b obtains total M_r^2 independent linear combinations of the $(M_t + M_c)M_r$ desired variables during M_r time slots.

Second, we need $M_t + M_c - M_r$ time slots. At time index $t \in \{2M_r + 1, \dots, M_t + M_c + M_r\}$, Transmitter a and Transmitter b transmit $X_{a,t} = [Y_{b[t-2M_r],1}, \dots, Y_{b[t-2M_r],M_r}, 0, \dots, 0]^T$ and $X_{b,t} = [Y_{a[t-2M_r],M_r+1}, \dots, Y_{a[t-2M_r],2M_r}, 0, \dots, 0]^T$, respectively, using only M_r antennas. The cognitive relay does not transmit. Since Receiver a knows $Y_{a[t-2M_r],M_r+1}, \dots, Y_{a[t-2M_r],2M_r}$ and Receiver b knows $Y_{b[t-2M_r],1}, \dots, Y_{b[t-2M_r],M_r}$ where $t \in \{2M_r + 1, \dots, M_t + M_c + M_r\}$, each receiver can eliminate interference terms and obtain $(M_t + M_c - M_r)M_r$ linearly independent interference-free signals during $M_t + M_c - M_r$ time slots. Therefore, the receivers have total $(M_t + M_c)M_r$ linearly independent equations involving $(M_t + M_c)M_r$ symbols, respectively. Consequently, we can obtain the $\left(\frac{(M_t+M_c)M_r}{M_t+M_c+M_r}, \frac{(M_t+M_c)M_r}{M_t+M_c+M_r}\right)$ DoF pair. The other points on the outer bound are achievable via time sharing.

- 4) Condition IV: $M_r < \frac{M_t+M_c}{2}$ and $M_r > M_t$

The DoF outer bound is determined by $\frac{d_a}{2M_r} + \frac{d_b}{M_r} \leq 1$ and $\frac{d_a}{M_r} + \frac{d_b}{2M_r} \leq 1$. We show that the $(\frac{2M_r}{3}, \frac{2M_r}{3})$ DoF pair on the DoF outer bound is achievable. All transmissions are scaled to satisfy the power constraint. First, we spend two time slots. At time index $t = 1$, Transmitter a sends M_t random linear combinations of $2M_r$ symbols, and the cognitive relay sends $2M_r - M_t$ different random linear combinations of the $2M_r$ symbols. The received signals at time index $t = 1$ can be represented as

$$Y_{a,1} = \mathbf{H}_{aa,1} \mathbf{U}_{a,1} S_a + \mathbf{H}_{ac,1} \mathbf{V}_{c,1} S_a \quad (14a)$$

$$Y_{b,1} = \mathbf{H}_{ba,1} \mathbf{U}_{a,1} S_a + \mathbf{H}_{bc,1} \mathbf{V}_{c,1} S_a, \quad (14b)$$

where $\mathbf{U}_{a,1}$ is a randomly chosen $M_t \times (2M_r)$ matrix with full rank, $\mathbf{V}_{c,1}$ is a randomly chosen $M_c \times (2M_r)$ matrix with rank M_r which includes a $(M_c - M_r) \times (2M_r)$ zero matrix, S_a is a $(2M_r) \times 1$ symbol vector for Receiver a , and noise terms are omitted. Receiver a has M_r linear combinations of intended $2M_r$ variables. Similarly, at time index $t = 2$, Transmitter b sends M_t random linear combinations of $2M_r$ symbols intended for Receiver b , and the cognitive relay sends different $2M_r - M_t$ random linear combinations of the $2M_r$ symbols. The received signals at time index $t = 2$ are

$$Y_{a,2} = \mathbf{H}_{ab,2} \mathbf{U}_{b,2} Q_b + \mathbf{H}_{ac,2} \mathbf{V}_{c,2} Q_b, \quad (15a)$$

$$Y_{b,2} = \mathbf{H}_{bb,2} \mathbf{U}_{b,2} Q_b + \mathbf{H}_{bc,2} \mathbf{V}_{c,2} Q_b, \quad (15b)$$

where $\mathbf{U}_{b,2}$ is a randomly chosen $M_t \times (2M_r)$ matrix with full rank, $\mathbf{V}_{c,2}$ is a randomly chosen $M_c \times (2M_r)$ matrix with rank M_r which includes a $(M_c - M_r) \times (2M_r)$ zero matrix, and Q_b is a $(2M_r) \times 1$ symbol vector for Receiver b . Receiver b obtains M_r linear combinations of intended $2M_r$ variables.

Second, we need one time slot indexed by $t = 3$. Transmitter a , Transmitter b , and the cognitive relay transmit $X_{a,3} = [Y_{b[1],1}, \dots, Y_{b[M_t],1}]^T$, $X_{b,3} = [Y_{a[1],2}, \dots, Y_{a[M_t],2}]^T$ and $X_{c,3} = [Y_{b[M_t+1],1} + Y_{a[M_t+1],2}, \dots, Y_{b[M_r],1} + Y_{a[M_r],2}, 0, \dots, 0]^T$, respectively, using delayed CSI. At time index $t = 3$, the received signals are

$$Y_{a,3} = \mathbf{H}_{aa,3} X_{a,3} + \mathbf{H}_{ab,3} X_{b,3} + \mathbf{H}_{ac,3} X_{c,3}, \quad (16a)$$

$$Y_{b,3} = \mathbf{H}_{ba,3} X_{a,3} + \mathbf{H}_{bb,3} X_{b,3} + \mathbf{H}_{bc,3} X_{c,3}. \quad (16b)$$

Since the interference terms at each receiver are comprised of the received signals in the previous time slots, each receiver can obtain M_r linearly independent interference-free signals at $t = 3$. Thus, each receiver has total $2M_r$ linearly independent equations involving $2M_r$ symbols and thus

we can obtain the $(\frac{2M_r}{3}, \frac{2M_r}{3})$ DoF pair which is the same achievable DoF pair in Condition V.

We can achieve all points on the DoF outer bound via time sharing.

5) Condition V: $M_r < \frac{M_t + M_c}{2}$ and $M_r \leq M_t$

In this case a DoF outer bound is given by $\frac{d_a}{2M_r} + \frac{d_b}{M_r} \leq 1$ and $\frac{d_a}{M_r} + \frac{d_b}{2M_r} \leq 1$. We show that the $(\frac{2M_r}{3}, \frac{2M_r}{3})$ DoF pair on the DoF outer bound is achievable.

At time index $t = 1$, if $2M_r > M_t$, Transmitter a sends M_t random linear combinations of $2M_r$ symbols with M_t transmit antennas, and the cognitive relay sends $2M_r - M_t$ different random linear combinations of the $2M_r$ symbols. If $2M_r \leq M_t$, then Transmitter a only transmits and the cognitive relay is silent. The received signals at time index $t = 1$ can be represented as (14a) and (14b) where $\mathbf{U}_{a,1}$ is a randomly chosen $M_t \times 2M_r$ matrix with full rank, $\mathbf{V}_{c,1}$ is a randomly chosen $M_c \times 2M_r$ matrix with rank $(2M_r - M_t)^+$ which includes a $(M_c - (2M_r - M_t)^+) \times 2M_r$ zero matrix, $(x)^+ = \max(x, 0)$, S_a is a $2M_r \times 1$ symbol vector for Receiver a , and noise terms are omitted. Receiver a has M_r linear combinations of intended $2M_r$ variables. Similarly, at time index $t = 2$, Transmitter b sends M_t random linear combinations of $2M_r$ symbols intended for Receiver b , and the cognitive relay sends different $2M_r - M_t$ random linear combinations of the $2M_r$ symbols if $2M_r > M_t$. If $2M_r \leq M_t$, then Transmitter b only transmits and the cognitive relay becomes silent. The received signals at time index $t = 2$ are the same as (15a) and (15b) where $\mathbf{U}_{b,2}$ is a randomly chosen $M_t \times 2M_r$ matrix with full rank, $\mathbf{V}_{c,2}$ is a randomly chosen $M_c \times 2M_r$ random matrix with rank $(2M_r - M_t)^+$ which includes a $(M_c - (2M_r - M)^+) \times 2M_r$ zero matrix, and Q_b is a $2M_r \times 1$ symbol vector for Receiver b . Receiver b obtains M_r linear combinations of the intended $2M_r$ variables.

At time index $t = 3$, Transmitter a and Transmitter b only transmit $X_{a,3} = [Y_{b[1],1}, \dots, Y_{b[M_r],1}, 0, \dots, 0]^T$ and $X_{b,3} = [Y_{a[1],2}, \dots, Y_{a[M_r],2}, 0, \dots, 0]^T$, respectively, using M_r antennas, and the cognitive relay does not transmit. Since the interference signals at each receiver at $t = 3$ are the received signals in previous time slots, each receiver can obtain M_r linearly independent

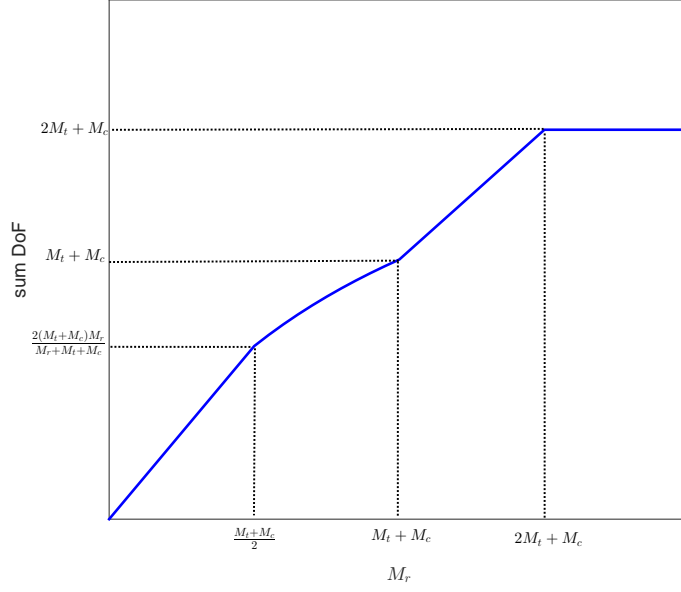


Fig. 5. The sum DoF of the MIMO Gaussian ICCR with delayed CSIT for fixed M_t and M_c

interference-free signals at $t = 3$. Thus, each receiver has total $2M_r$ linearly independent equations involving $2M_r$ symbols and hence we can obtain the $(\frac{2M_r}{3}, \frac{2M_r}{3})$ DoF pair on the DoF outer bound. The other points on the outer bound are achievable via time sharing.

■

Remark 5: Fig. 5 shows the result of Theorem 4 in terms of sum DoF for fixed M_t and M_c . For Condition I, III, and V, the cognitive relay does not utilize delayed feedback information. In other words, except when $M_t < M_r < M_t + M_c$, the optimal DoF region can be obtained regardless of the availability of delayed feedback information at the cognitive relay. This optimal DoF region will be again addressed in Section V.

B. Delayed Output Feedback

In this case each transmitter knows the output of the intended receiver after one time slot delay, and the cognitive relay has the output feedback from both receivers after one time slot delay. The DoF region

is characterized as follows.

Corollary 2: The DoF region of the MIMO ICCR with delayed output feedback is

$$\mathcal{D}_{\text{output}} = \mathcal{D}_{\text{CSI}}. \quad (17)$$

Proof: For Condition I, the outer bound is achievable similar to the case of delayed CSIT, since the related scheme does not exploit any delayed feedback information. For Conditions II, III, IV, and V, the achievable scheme is an extension of the SISO scheme using delayed output feedback, which has three parts. First, Transmitter a and cognitive relay send messages of Receiver a . Second, Transmitter b and cognitive relay transmit messages for Receiver b during different time slots as in the scheme for delayed CSIT. Third, the transmitters and cognitive relay transmit the outputs fed back from the receivers in previous time slots, instead of transmitting linear combinations of the past symbols with delayed CSI. Then, similar to the delayed CSIT case, the receivers can eliminate interference terms since the interference signals are already known at each receiver. Thus, the DoF region with delayed output feedback is the same as that of the delayed CSIT case. ■

C. Delayed Shannon Feedback

Corollary 3: The DoF region of the MIMO ICCR with delayed Shannon feedback is

$$\mathcal{D}_{\text{Shannon}} = \mathcal{D}_{\text{CSI}}. \quad (18)$$

Proof: Since the DoF outer bounds with delayed Shannon feedback are identical to those with delayed CSIT or delayed output feedback, the same optimal DoF region can be obtained with the scheme utilizing delayed CSIT or output feedback information. The outer bound will be proved in Section VI. ■

D. No Feedback

Corollary 4: The DoF region of the MIMO ICCR with no feedback is

$$\mathcal{D}_{\text{no}} = \left\{ (d_a, d_b) \in \mathbb{R}_+^2 : d_a \leq \min(M_t + M_c, M_r), d_b \leq \min(M_t + M_c, M_r), \right.$$

$$d_a + d_b \leq \min(2M_t + M_c, M_r) \} \quad (19)$$

where M_t , M_r , and M_c are the numbers of antennas at the transmitter, the receiver and the cognitive relay, respectively.

Proof: We show that the outer bound that will be presented in Corollary 5 of Section VI is achievable.

We consider the following three conditions:

- $2M_t + M_c \leq M_r$
- $M_t + M_c \leq M_r < 2M_t + M_c$
- $M_r < M_t + M_c$

If $2M_t + M_c \leq M_r$ or $M_t + M_c \leq M_r < 2M_t + M_c$, the optimal scheme is the same as that for the delayed CSIT. This is because in Theorem 4, the proposed scheme for the delayed CSIT in Condition I is optimal but does not use any delayed feedback information so that the optimal DoF region can be obtained by this scheme. Therefore, if $2M_t + M_c \leq M_r$, the DoF region is determined by $d_a \leq M_t + M_c$, $d_b \leq M_t + M_c$, and $d_a + d_b \leq 2M_t + M_c$. If $M_t + M_c \leq M_r < 2M_t + M_c$, the DoF region is determined by $d_a \leq M_t + M_c$, $d_b \leq M_t + M_c$, and $d_a + d_b \leq M_r$. On the other hand, if $M_r < M_t + M_c$, the DoF outer bound $d_a + d_b \leq M_r$ is achievable via TDM, similar to the result for the SISO case in Corollary 1.

We note this corollary can also be obtained using the result of MIMO BC without CSIT (i.e., without feedback) in [26], [27]. ■

Remark 6: If $M_t + M_c \leq M_r$ (i.e., Condition I), the DoF region with delayed feedback in Theorem 4 is the same as that with no feedback. This result indicates that neither of the three types of delayed feedback information are useful when $M_t + M_c \leq M_r$. Therefore, delayed feedback information is useful in terms of DoF if $M_r < M_t + M_c$. Fig. 6 shows the improvements of the DoF region by delayed feedback information at both the transmitters and the cognitive relay, when $M_r < \frac{M_t + M_c}{2}$ and $\frac{M_t + M_c}{2} \leq M_r < M_t + M_c$, compared to the case of no feedback.

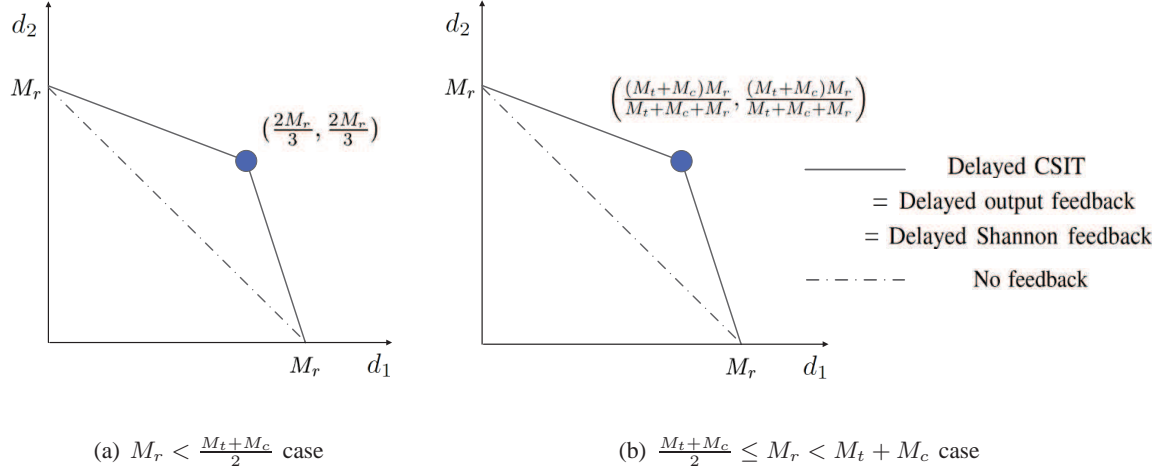


Fig. 6. The DoF region of the MIMO Gaussian ICCR with and without delayed feedback at both transmitters and cognitive relay.

V. ACHIEVABLE DOF WITH DELAYED FEEDBACK UNAVAILABLE AT COGNITIVE RELAY

In this section, we consider the case when the transmitters have delayed feedback information but the cognitive relay does not. Using a similar approach in Section IV, We derive the achievable DoF region.

Theorem 5: When the cognitive relay does not have delayed feedback information, the DoF region of the MIMO ICCR achieved by the proposed retrospective interference alignment is

$$\mathcal{D}'_{\text{delay} \setminus \text{CR}} \subset \bar{\mathcal{D}}_{\text{delay}}, \quad \text{if } M_t < M_r < M_t + M_c,$$

$$\mathcal{D}'_{\text{delay} \setminus \text{CR}} = \bar{\mathcal{D}}_{\text{delay}}, \quad \text{otherwise,}$$

where M_t , M_r , and M_c are the numbers of antennas at the transmitter, the receiver and the cognitive relay, respectively.

Proof: The DoF outer bound that will be derived in Theorem 6 of Section VI is also valid when delayed feedback information is not available at the cognitive relay. We already showed that with delayed feedback, the DoF region is achieved under Conditions I, III, and V even if the cognitive relay does not have any feedback information. Thus, we consider only the two cases of Conditions II and IV.

With delayed CSIT under Condition II, a DoF outer bound with delayed CSIT for this case is given by $\frac{d_a}{M_t+M_c} + \frac{d_b}{M_r} \leq 1$ and $\frac{d_a}{M_r} + \frac{d_b}{M_t+M_c} \leq 1$. If $2M_t \geq M_r$, we can show that the $\left(\frac{(M_t+M_c)M_t}{3M_t+M_c-M_r}, \frac{(M_t+M_c)M_t}{3M_t+M_c-M_r}\right)$ DoF pair is achievable but it does not meet the DoF outer bound. All transmissions are scaled to satisfy the power constraint. First, we spend $2M_t$ time slots. At each time index $t \in \{1, \dots, M_t\}$, Transmitter a sends random linear combinations of $M_t + M_c$ independent symbols, and the cognitive relay sends distinct random linear combinations of the $M_t + M_c$ independent symbols. Thus, Receiver a has $M_r M_t$ independent linear equations of the $(M_t + M_c)M_t$ desired symbols. At each time index $t \in \{M_t + 1, \dots, 2M_t\}$, Transmitter b and the cognitive relay send distinct random linear combinations of $M_t + M_c$ symbols intended for Receiver b . Receiver b obtains $M_r M_t$ independent linear combinations of the $(M_t + M_c)M_t$ desired variables. Finally, we need another $M_t + M_c - M_r$ time slots $t \in \{2M_t + 1, \dots, 3M_t + M_c - M_r\}$, when Transmitter a and Transmitter b transmit $X_{a,t} = [Y_{b[t-2M_t],1}, \dots, Y_{b[t-2M_t],M_t}]^T$ and $X_{b,t} = [Y_{a[t-2M_t],M_t+1}, \dots, Y_{a[t-2M_t],2M_t}]^T$, respectively, using delayed CSI, but the cognitive relay is silent. Since the interfering terms are comprised of the past received signal in previous slots, each receiver can eliminate the interfering terms and obtain $(M_t + M_c - M_r)M_t$ linearly independent interference-free signals at $t \in \{2M_t + 1, \dots, 3M_t + M_c - M_r\}$. At the end of transmission, each receiver has total $(M_t + M_c)M_t (= M_r M_t + (M_t + M_c - M_r)M_t)$ linearly independent equations involving $(M_t + M_c)M_t$ symbols during $3M_t + M_c - M_r (= 2M_t + (M_t + M_c - M_r))$ time slots. Therefore, we can obtain the $\left(\frac{(M_t+M_c)M_t}{3M_t+M_c-M_r}, \frac{(M_t+M_c)M_t}{3M_t+M_c-M_r}\right)$ DoF pair, and total $\frac{2(M_t+M_c)M_t}{3M_t+M_c-M_r}$ DoF when $2M_t \geq M_r$. If $2M_t < M_r$, the sum DoF $\frac{2(M_t+M_c)M_t}{3M_t+M_c-M_r}$ achieved by the proposed scheme is less than M_r , but M_r is achievable via time sharing. Thus, if you adopt time sharing instead of the proposed scheme when $2M_t < M_r$, total M_r DoF can be achievable. The other points on the boundary of the achievable region can be obtained via time sharing.

For Condition IV (i.e., $M_r < \frac{M_t+M_c}{2}$ and $M_r > M_t$), we can show that the $\left(\frac{M_t+M_r}{3}, \frac{M_t+M_r}{3}\right)$ DoF pair is achievable, but it is below the outer bound determined by $\frac{d_a}{2M_r} + \frac{d_b}{M_r} \leq 1$ and $\frac{d_a}{M_r} + \frac{d_b}{2M_r} \leq 1$ if $2M_t \leq M_r$. All transmissions are scaled to satisfy the power constraint. First, we spend two time slots. At time

index $t = 1$, Transmitter a sends M_t random linear combinations of $M_t + M_r$ symbols with M_t transmit antennas, and the cognitive relay sends M_r different random linear combinations of the $M_t + M_r$ symbols with M_r antennas. Similarly, at time index $t = 2$, Transmitter b sends M_t random linear combinations of $M_t + M_r$ symbols intended for Receiver b , and the cognitive relay sends different M_r random linear combinations of the $M_t + M_r$ symbols. Receiver b obtains M_r linear combinations of intended $M_t + M_r$ variables. Second, we need one time slot indexed by $t = 3$. Transmitter a and Transmitter b only transmit $X_{a,3} = [Y_{b[1],1}, \dots, Y_{b[M_t],1}]^T$ and $X_{b,3} = [Y_{a[1],2}, \dots, Y_{a[M_t],2}]^T$, respectively, using delayed CSI, and the cognitive relay does not transmit. Since the interference terms at each receiver are comprised of the received signals in the previous time slots, each receiver can obtain M_t linearly independent interference-free signals at $t = 3$. Thus, each receiver has total $M_t + M_r$ linearly independent equations involving $M_t + M_r$ symbols so that we can obtain the $(\frac{M_t+M_r}{3}, \frac{M_t+M_r}{3})$ DoF pair and total $\frac{2(M_t+M_r)}{3}$ DoF. If $2M_t < M_r$, the sum DoF $\frac{2(M_t+M_r)}{3}$ achieved by the proposed scheme is less than M_r , but M_r can be achieved by time sharing. Therefore, we can adopt time sharing if $2M_t < M_r$ instead of the proposed scheme. Then, the achievable sum DoF is M_r when $2M_t < M_r$. The other points on the boundary of the achievable region are achievable via time sharing.

Similarly, the DoF region of the ICCR with delayed output feedback can be obtained for Condition II and IV. In the second part, the transmitters send the outputs fed back from the receivers instead of using delayed CSI. Then, we can obtain the same DoF region as that with delayed CSIT. Since Shannon feedback includes CSI and output feedback and the achievable DoF regions with delayed CSIT and delayed feedback are identical, with delayed Shannon feedback, the same achievable DoF region can be obtained by the scheme utilizing delayed CSIT or output feedback information. ■

Remark 7: For Condition II and IV, i.e., $M_t < M_r < M_t + M_c$, the achievable DoF pairs do not meet the outer bound when the delayed feedback information is not available at the cognitive relay. Therefore, the proposed retrospective scheme is optimal except when $M_t < M_r < M_t + M_c$, in which case no statement about optimality can be made at this time.

Remark 8: Comparing Theorem 5 with Corollary 4, delayed feedback information at only transmitters is useful in terms of DoF only if $\frac{M_t+M_c}{2} \leq M_r < \min(M_t + M_c, 2M_t)$ or $M_r < \min(\frac{M_t+M_c}{2}, 2M_t)$.

Remark 9: Fig. 7 shows the achievable sum DoFs for the two cases with/without delayed feedback at the cognitive relay when $M_r > M_t$ for fixed M_t and M_c . Fig. 7(a) corresponds to Condition I and II when $\frac{M_t+M_c}{2} \leq M_t$, and Fig. 7(b) corresponds to Condition I, II and IV when $M_t < \frac{M_t+M_c}{2} < 2M_t$.

VI. DOF OUTER BOUNDS

A. Delayed feedback

The following outer bound holds for all three types of feedback information discussed in this paper.

Theorem 6: The DoF region with delayed feedback is contained in the following region:

$$\bar{\mathcal{D}}_{\text{delayed}} = \left\{ (d_a, d_b) \in \mathbb{R}_+^2 : \begin{aligned} & d_a \leq \min(M_r, M_t + M_c), \quad d_b \leq \min(M_r, M_t + M_c), \\ & \frac{d_a}{\min(M_r, M_t + M_c)} + \frac{d_b}{\min(2M_r, M_t + M_c)} \leq \frac{\min(M_r, 2M_t + M_c)}{\min(M_r, M_t + M_c)}, \\ & \frac{d_a}{\min(2M_r, M_t + M_c)} + \frac{d_b}{\min(M_r, M_t + M_c)} \leq \frac{\min(M_r, 2M_t + M_c)}{\min(M_r, M_t + M_c)} \end{aligned} \right\}$$

where M_t , M_r , and M_c are the numbers of antennas at the transmitter, the receiver and the cognitive relay, respectively.

Lemma 1: For a given $t \in \{1, 2, \dots, n\}$,

$$\frac{1}{c_1} h(Y_{a[1:c_1],t} | Y_{a[1:M_r]}^{t-1}, W_a, \mathcal{H}^n) \geq \frac{1}{c_2} h(Y_{a[1:c_1],t}, Y_{b[1:c_2-c_1],t} | Y_{a[1:M_r]}^{t-1}, Y_{b[1:M_r]}^{t-1}, W_a, \mathcal{H}^n).$$

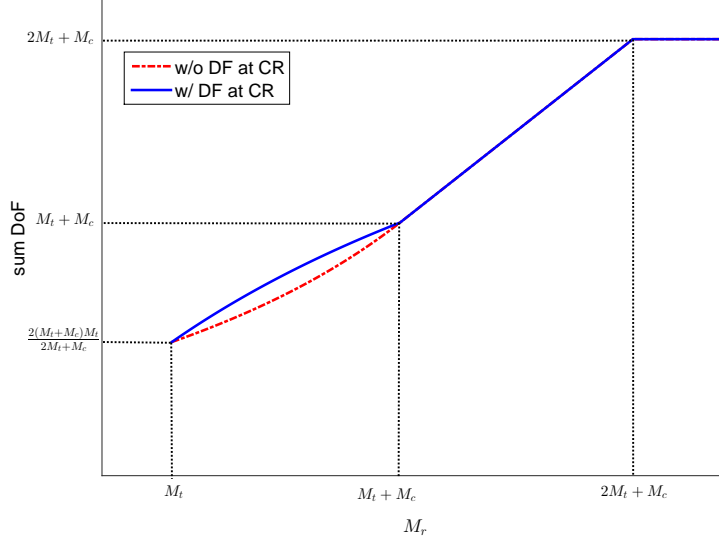
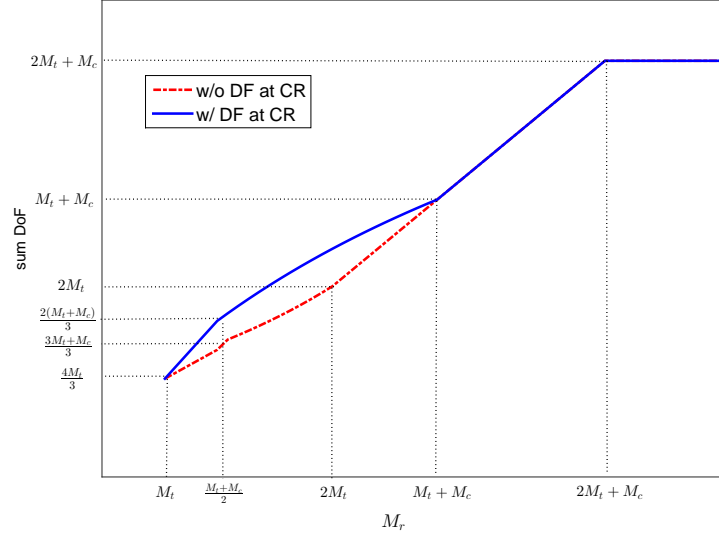
where $c_1 \triangleq \min(M_r, M_t + M_c)$ and $c_2 \triangleq \min(2M_r, M_t + M_c)$.

Proof: The key idea of this proof is the statistical equivalence of channel outputs [22], [23]. The detailed proof is in Appendix A. ■

Lemma 2: For the ICCR with delayed feedback information, we have

$$\frac{1}{\min(M_r, M_t + M_c)} h(Y_{a[1:M_r]}^n | W_a, \mathcal{H}^n) \geq \frac{1}{\min(2M_r, M_t + M_c)} h(Y_{a[1:M_r]}^n, Y_{b[1:M_r]}^n | W_a, \mathcal{H}^n) + n \cdot o(\log_2 P).$$

Proof: We use Lemma 1 to prove Lemma 2. The detailed proof is in Appendix B. ■

(a) $\frac{M_t+M_c}{2} \leq M_t$ (b) $M_t < \frac{M_t+M_c}{2} < 2M_t$ Fig. 7. The achievable sum DoF when $M_r > M_t$ for fixed M_t and M_c .

Using Lemma 1 and Lemma 2, we now prove Theorem 6.

Proof: The outer bounds $d_a \leq \min(M_r, M_t + M_c)$ and $d_b \leq \min(M_r, M_t + M_c)$ can be readily obtained from the numbers of antennas. The other bounds are obtained using the fact that the conditional distributions of $Y_{a[\ell_1],t}$ and $Y_{b[\ell_2],t}$ for all $\ell_1, \ell_2 \in \{1, \dots, M_r\}$ are identical when the two variables are conditioned on the collection of channel gains over all time, past channel outputs, and some present channel outputs.

For block length n , using Fano's inequality we can bound the rate R_a as

$$\begin{aligned} nR_a &\leq I(W_a; Y_{a[1:M_r]}^n | \mathcal{H}^n) + n\varepsilon_{a,n} \\ &= h(Y_{a[1:M_r]}^n | \mathcal{H}^n) - h(Y_{a[1:M_r]}^n | W_a, \mathcal{H}^n) + n\varepsilon_{a,n} \\ &\leq n \min(M_r, 2M_t + M_c) \log_2 P - h(Y_{a[1:M_r]}^n | W_a, \mathcal{H}^n) + n \cdot o(\log_2 P) + n\varepsilon_{a,n} \end{aligned} \quad (20)$$

where $\varepsilon_{a,n} \rightarrow 0$ as $n \rightarrow 0$.

For the rate R_b , we obtain an outer bound using Fano's inequality as

$$\begin{aligned} nR_b &\leq I(W_b; Y_{b[1:M_r]}^n | \mathcal{H}^n) + n\varepsilon_{b,n} \\ &\leq I(W_b; Y_{b[1:M_r]}^n, Y_{a[1:M_r]}^n | W_a, \mathcal{H}^n) + n\varepsilon_{b,n} \\ &= h(Y_{a[1:M_r]}^n, Y_{b[1:M_r]}^n | W_a, \mathcal{H}^n) - h(Y_{a[1:M_r]}^n, Y_{b[1:M_r]}^n | W_a, W_b, \mathcal{H}^n) + n\varepsilon_{b,n} \\ &\leq h(Y_{a[1:M_r]}^n, Y_{b[1:M_r]}^n | W_a, \mathcal{H}^n) + n\varepsilon_{b,n} \end{aligned} \quad (21)$$

where $\varepsilon_{b,n} \rightarrow 0$ as $n \rightarrow 0$.

By Lemma 2, we can combine (20) and (21) as

$$\frac{nR_a}{\min(M_r, M_t + M_c)} + \frac{nR_b}{\min(2M_r, M_t + M_c)} \leq \frac{n \min(M_r, 2M_t + M_c)}{\min(M_r, M_t + M_c)} \log_2 P + n \cdot o(\log_2 P) + n\varepsilon_n$$

where $\varepsilon_n = \varepsilon_{a,n} + \varepsilon_{b,n} \rightarrow 0$ as $n \rightarrow 0$. Hence, we obtain DoF outerbound as

$$\frac{d_a}{\min(M_r, M_t + M_c)} + \frac{d_b}{\min(2M_r, M_t + M_c)} \leq \frac{\min(M_r, 2M_t + M_c)}{\min(M_r, M_t + M_c)}.$$

Similarly, we can obtain

$$\frac{d_a}{\min(2M_r, M_t + M_c)} + \frac{d_b}{\min(M_r, M_t + M_c)} \leq \frac{\min(M_r, 2M_t + M_c)}{\min(M_r, M_t + M_c)}$$

by switching the receiver order. ■

B. No feedback

A DoF outer bound in the absence of CSIT can be obtained in a straight forward manner using the results from [26], [27].

Corollary 5: The outer bound of the DoF region with no feedback $\bar{\mathcal{D}}_{no}$ is

$$\bar{\mathcal{D}}_{no} = \left\{ (d_a, d_b) \in \mathbb{R}_+^2 : d_a \leq \min(M_t + M_c, M_r), d_b \leq \min(M_t + M_c, M_r), \right. \\ \left. d_a + d_b \leq \min(2M_t + M_c, M_r) \right\}. \quad (22)$$

where M_t , M_r , and M_c are the numbers of antennas at the transmitter, the receiver, and the cognitive relay, respectively.

Proof: Consider a transmitter-cooperative outer bound. Because the transmitter cooperation results in the MIMO broadcast channel with $2M_t + M_c$ transmit antennas and two receivers with M_r -antenna each. The DoF outer bound follows directly from the results of [26], [27]. ■

VII. DISCUSSIONS

A. Comparisons with Broadcast and Interference Channel with delayed CSIT

If cooperation among transmitters and cognitive relay is allowed, the ICCR becomes equivalent to the broadcast channel where the transmitter has $2M_t + M_c$ antennas and each receiver has M_r antennas. Therefore, when CSIT is delayed, the DoF region of the broadcast channel with antenna configuration $(2M_t + M_c, M_r, M_r)$ is a superset of the DoF of the ICCR under $(M_t, M_t, M_c, M_r, M_r)$. Table III shows a comparison of the sum-DoF under delayed CSIT between a broadcast channel [16] and ICCR where delayed CSIT is available at all nodes. If $2M_t + M_c \leq M_r$ or $M_r < \frac{M_t + M_c}{2}$, the sum DoF is the same.

TABLE III
SUM DoFs FOR MIMO BROADCAST CHANNEL, ICCR AND INTERFERENCE CHANNEL WITH DELAYED CSIT

	broadcast channel [16]	ICCR	interference channel (for even M_c) [22]
$2M_t + M_c \leq M_r$	$2M_t + M_c$	$2M_t + M_c$	$2M_t + M_c$
$M_t + M_c \leq M_r < 2M_t + M_c$	$\frac{2(2M_t + M_c)M_r}{2M_t + M_c + M_r}$	M_r	M_r
$M_t + \frac{M_c}{2} \leq M_r < M_t + M_c$	$\frac{2(2M_t + M_c)M_r}{2M_t + M_c + M_r}$	$\frac{2(M_t + M_c)M_r}{M_t + M_c + M_r}$	M_r
$\frac{M_t + M_c}{2} \leq M_r < M_t + \frac{M_c}{2}$	$\frac{4M_r}{3}$	$\frac{2(M_t + M_c)M_r}{M_t + M_c + M_r}$	$\frac{2(M_t + \frac{M_c}{2})M_r}{M_t + \frac{M_c}{2} + M_r}$
$M_r < \frac{M_t + M_c}{2}$	$\frac{4M_r}{3}$	$\frac{4M_r}{3}$	$\frac{4M_r}{3}$

For the other scenarios (i.e., $M_t + \frac{M_c}{2} < M_r < 2M_t + M_c$), the sum DoF of the MIMO ICCR with delayed CSIT is less than that of the MIMO broadcast channel.

Table III reproduces from [22] the sum-DoF of the MIMO interference channel, with $M_t + \frac{M_c}{2}$ transmit and M_r receive antennas at respective nodes. If the two transmitters partially cooperate, the channel becomes equivalent to the ICCR with antenna configuration of $(M_t, M_t, M_c, M_r, M_r)$. Therefore, the DoF region of the interference channel with delayed CSIT is included by that of the ICCR with delayed CSIT. If $M_t + M_c \leq M_r$ or $M_r < \frac{M_t + M_c}{2}$, the sum DoF is the same for both channels. If $\frac{M_t + M_c}{2} \leq M_r \leq M_t + M_c$, however, the sum DoF of the ICCR is greater than that of the interference channel because the cognitive relay effectively produces partial cooperation between transmitters.

B. Extension to Cognitive Interference Channel

Here we consider another extension to the cognitive interference channel consisting of one non-cognitive transmitter, one cognitive transmitter, and their intended receivers. The cognitive transmitter has both messages intended for the two receivers as shown in Fig. 8. The cognitive interference channel with perfect CSIT and CSIR has been studied in [29]–[34]. The inner and outer bounds of capacity region of the SISO cognitive interference channel with perfect CSIT were given in [29]–[31]. For MIMO cognitive interference channel, [32], [33] calculated the capacity region within a constant gap. In [34],

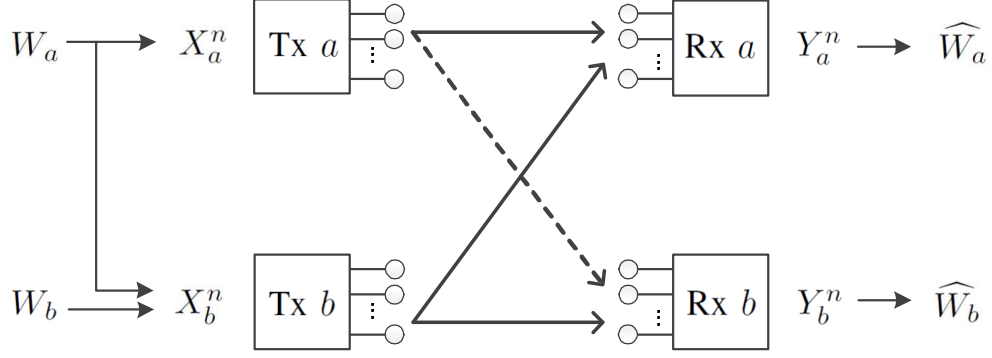


Fig. 8. A MIMO cognitive interference channel.

the DoF region of the cognitive interference channel with perfect CSIT was derived. However, the DoF region of the cognitive interference channel with delayed feedback has been unknown. The DoF of the ICCR from the previous section can be used for a lower and an upper bound of the cognitive interference channel when feedback is delayed.

Corollary 6: The DoF region of the cognitive interference channel with antenna configuration $(M_t, M_t + M_c, M_r, M_r)$ is lower bounded by that of the ICCR with antenna configuration $(M_t, M_t, M_c, M_r, M_r)$.

Proof: If cooperation between the cognitive relay and one transmitter is allowed in the ICCR, the channel becomes equivalent to the cognitive interference channel where the non-cognitive transmitter has M_t antennas, the cognitive transmitter has $M_t + M_c$ antennas, and each receiver has M_r antennas. Therefore, the DoF region of the cognitive interference channel with antenna configuration $(M_t, M_t + M_c, M_r, M_r)$ is lower bounded by that of the ICCR with antenna configuration $(M_t, M_t, M_c, M_r, M_r)$. ■

Corollary 7: The DoF region of the cognitive interference channel with antenna configuration (M_t, M_c, M_r, M_r) is upper bounded by the DoF region of the ICCR with antenna configuration $(M_t, M_t, M_c, M_r, M_r)$.

Proof: If only one transmitter exists in the ICCR, the antenna configuration for this scenario is $(M_t, 0, M_c, M_r, M_r)$ and it corresponds to the cognitive interference channel where the non-cognitive transmitter has M_t antennas and the cognitive transmitter has M_c antennas while each receiver has M_r an-

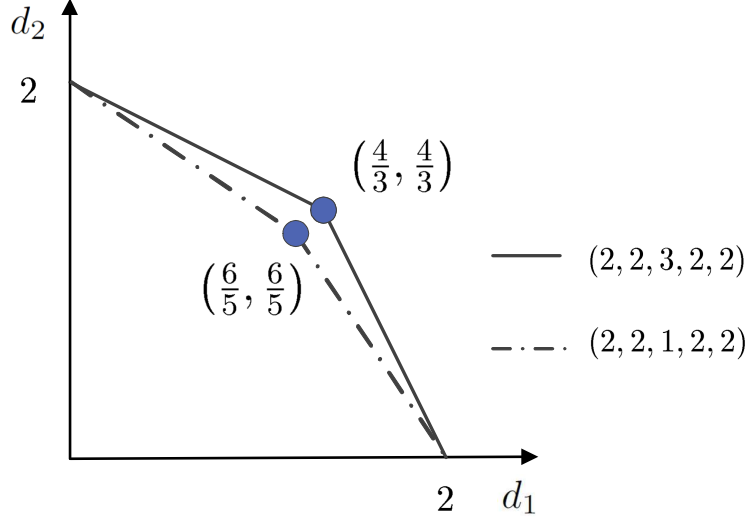


Fig. 9. The DoF regions of the ICCR with delayed feedback for $(2, 2, 1, 2, 2)$ and $(2, 2, 3, 2, 2)$.

tennas. Hence, the upper bound of the DoF region of the cognitive interference channel with antenna configuration of (M_t, M_c, M_r, M_r) is that of the ICCR with antenna configuration of $(M_t, M_t, M_c, M_r, M_r)$. ■

Example 1 (Cognitive interference channel with antenna configuration $(2, 3, 2, 2)$): In this example the non-cognitive transmitter has two antennas, the cognitive transmitter has three antennas, and receivers have two antennas each.

The DoF region of the $(2, 2, 1, 2, 2)$ ICCR can serve as a lower bound, and the DoF of $(2, 2, 3, 2, 2)$ ICCR serves as upper bound. The lower bound derived in Section IV is

$$\mathcal{D}_{\text{CSI}} = \left\{ (d_a, d_b) \in \mathbb{R}_+^2 : \frac{d_a}{2} + \frac{d_b}{3} \leq 1, \frac{d_a}{3} + \frac{d_b}{2} \leq 1 \right\}, \quad (23)$$

where the maximum sum DoF is $\frac{12}{5}$. The upper bound is obtained from the results of Section IV as

$$\mathcal{D}_{\text{CSI}} = \left\{ (d_a, d_b) \in \mathbb{R}_+^2 : \frac{d_a}{2} + \frac{d_b}{4} \leq 1, \frac{d_a}{4} + \frac{d_b}{2} \leq 1 \right\}, \quad (24)$$

where the maximum sum DoF is $\frac{8}{3}$. These lower and upper bounds are shown in Fig. 9.

VIII. CONCLUSION

This paper studies the DoF region of the two-user Gaussian fading interference channel with cognitive relay (ICCR) with delayed feedback. Three different types of delayed feedback are considered: delayed CSIT, delayed output feedback, and delayed Shannon feedback. For the SISO ICCR, the proposed retrospective interference alignment scheme using delayed feedback information achieves the DoF region. The sum DoF of the SISO ICCR is $4/3$ with delayed feedback information, compared to the DoF of 1 for SISO interference channel in the absence of relay, regardless of CSIT. Without feedback, the cognitive relay is not useful in the sense of DoF.

In the MIMO case, the optimal DoF has been characterized under all antenna configurations if delayed feedback is provided to both the transmitters and cognitive relay. DoF benefits can be obtained over and above the open-loop system when $\frac{M_t+M_c}{2} \leq M_r < M_t + M_c$ or $M_r < \frac{M_t+M_c}{2}$, and the sum DoF is $\frac{2(M_t+M_c)M_r}{M_t+M_c+M_r}$ when $\frac{M_t+M_c}{2} \leq M_r < M_t + M_c$ and $\frac{4M_r}{3}$ when $M_r < \frac{M_t+M_c}{2}$. On the other hand, if $2M_t + M_c \leq M_r$ or $M_t + M_c \leq M_r < 2M_t + M_c$, then delayed feedback does not help in the sense of DoF. In this scenario, the sum-DoF (for both open-loop and closed-loop) is $2M_t + M_c$ when $2M_t + M_c \leq M_r$ and M_r when $M_t + M_c \leq M_r < 2M_t + M_c$.

If delayed feedback is unavailable at the cognitive relay, the proposed retrospective interference alignment scheme achieves the optimal DoF except when $M_t < M_r < M_t + M_c$, where existing upper and lower bounds do not meet. Delayed feedback is shown to extend the DoF over and above the open-loop system when $(M_t + M_c)/2 \leq M_r < \min(M_t + M_c, 2M_t)$ and $M_r < \min((M_t + M_c)/2, 2M_t)$.

In addition, in this paper upper and lower bounds are derived for the DoF region of the two-user MIMO cognitive interference channel under delayed feedback.

APPENDIX A

PROOF OF LEMMA 1

$$\begin{aligned}
\frac{1}{c_1} h(Y_{a[1:c_1],t} | Y_{a[1:M_r]}^{t-1}, W_a, \mathcal{H}^n) &= \frac{1}{c_1} \sum_{\ell=1}^{c_1} h(Y_{a[\ell],t} | Y_{a[1:\ell-1],t}, Y_{a[1:M_r]}^{t-1}, W_a, \mathcal{H}^n) \\
&\stackrel{(a)}{\geq} \frac{1}{c_1} \sum_{\ell=1}^{c_1} h(Y_{a[\ell],t} | Y_{a[1:c_1-1],t}, Y_{a[1:M_r]}^{t-1}, W_a, \mathcal{H}^n) \tag{A.1}
\end{aligned}$$

where (a) holds because conditioning reduces entropy. Since $X_{a,t} = f_{a,t}(W_a, \mathcal{H}^{t-1})$ if only delayed CSIT is available, we can write

$$\begin{aligned}
h(Y_{a[\ell_1],t} | Y_{a[1:c_1-1],t}, Y_{a[1:M_r]}^{t-1}, W_a, \mathcal{H}^n) &= h(Y_{a[\ell_1],t} | X_{a,t}, Y_{a[1:c_1-1],t}, Y_{a[1:M_r]}^{t-1}, W_a, \mathcal{H}^n) \\
&= h(\tilde{Y}_{a[\ell_1],t} | Y_{a[1:c_1-1],t}, Y_{a[1:M_r]}^{t-1}, W_a, \mathcal{H}^n) \tag{A.2}
\end{aligned}$$

$$\begin{aligned}
h(Y_{b[\ell_2],t} | Y_{a[1:c_1-1],t}, Y_{a[1:M_r]}^{t-1}, W_a, \mathcal{H}^n) &= h(Y_{b[\ell_2],t} | X_{a,t}, Y_{a[1:c_1-1],t}, Y_{a[1:M_r]}^{t-1}, W_a, \mathcal{H}^n) \\
&= h(\tilde{Y}_{b[\ell_2],t} | Y_{a[1:c_1-1],t}, Y_{a[1:M_r]}^{t-1}, W_a, \mathcal{H}^n) \tag{A.3}
\end{aligned}$$

where $\tilde{Y}_{i[\ell],t}$ is defined as the output signal if we assume $X_{a,t} = \mathbf{0}_{M_t \times 1}$, i.e., in the absence of $X_{a,t}$. We note that $\tilde{Y}_{a[\ell_1],t}$ and $\tilde{Y}_{b[\ell_2],t}$ have identical distributions since the respective channel gains and receiver noises have identical distributions. Thus, for all ℓ_1 and ℓ_2 ,

$$h(\tilde{Y}_{a[\ell_1],t} | Y_{a[1:c_1-1],t}, Y_{a[1:M_r]}^{t-1}, W_a, \mathcal{H}^n) = h(\tilde{Y}_{b[\ell_2],t} | Y_{a[1:c_1-1],t}, Y_{a[1:M_r]}^{t-1}, W_a, \mathcal{H}^n) \tag{A.4}$$

therefore,

$$h(Y_{a[\ell_1],t} | Y_{a[1:c_1-1],t}, Y_{a[1:M_r]}^{t-1}, W_a, \mathcal{H}^n) = h(Y_{b[\ell_2],t} | Y_{a[1:c_1-1],t}, Y_{a[1:M_r]}^{t-1}, W_a, \mathcal{H}^n). \tag{A.5}$$

This represents the statistical equivalence of channel outputs [22], [23]. The result (A.5) is also applicable even if $X_{a,t} = f_{a,t}(W_a, \mathcal{H}^{t-1})$ is replaced by $X_{a,t} = f_{a,t}(W_a, Y_a^{t-1})$ or $X_{a,t} = f_{a,t}(W_a, Y_a^{t-1}, \mathcal{H}^{t-1})$ for the delayed feedback or the Shannon feedback information, since $X_{a,t}$ when either delayed feedback or Shannon feedback information is available can be constructed from the given conditions for the delayed

CSIT case. Thus, we can rewrite (A.1) as

$$\begin{aligned}
\frac{1}{c_1} h(Y_{a[1:c_1],t} | Y_{a[1:M_r]}^{t-1}, W_a, \mathcal{H}^n) &\geq \frac{1}{c_1} \sum_{\ell=1}^{c_1} h(Y_{a[\ell],t} | Y_{a[1:c_1-1],t}, Y_{a[1:M_r]}^{t-1}, W_a, \mathcal{H}^n) \\
&\stackrel{(b)}{=} h(Y_{b[1],t} | Y_{a[1:c_1-1],t}, Y_{a[1:M_r]}^{t-1}, W_a, \mathcal{H}^n) \\
&\stackrel{(c)}{\geq} h(Y_{b[1],t} | Y_{a[1:c_1],t}, Y_{a[1:M_r]}^{t-1}, W_a, \mathcal{H}^n) \\
&\stackrel{(d)}{=} \frac{1}{c_2 - c_1} \sum_{\ell=1}^{c_2 - c_1} h(Y_{b[\ell],t} | Y_{a[1:c_1],t}, Y_{a[1:M_r]}^{t-1}, W_a, \mathcal{H}^n) \\
&\stackrel{(e)}{\geq} \frac{1}{c_2 - c_1} \sum_{\ell=1}^{c_2 - c_1} h(Y_{b[\ell],t} | Y_{b[1:\ell-1],t}, Y_{a[1:c_1],t}, Y_{a[1:M_r]}^{t-1}, W_a, \mathcal{H}^n) \\
&= \frac{1}{c_2 - c_1} h(Y_{b[1:c_2-c_1],t} | Y_{a[1:c_1],t}, Y_{a[1:M_r]}^{t-1}, W_a, \mathcal{H}^n) \tag{A.6}
\end{aligned}$$

where (b) and (d) follow from the statistical equivalence of channel outputs, and (c) and (e) follow from the fact that conditioning reduces entropy. Thus, we have

$$\begin{aligned}
c_2 h(Y_{a[1:c_1],t} | Y_{a[1:M_r]}^{t-1}, W_a, \mathcal{H}^n) &\geq c_1 h(Y_{a[1:c_1],t}, Y_{b[1:c_2-c_1],t} | Y_{a[1:M_r]}^{t-1}, W_a, \mathcal{H}^n) \\
&\stackrel{(f)}{\geq} c_1 h(Y_{a[1:c_1],t}, Y_{b[1:c_2-c_1],t} | Y_{a[1:M_r]}^{t-1}, Y_{b[1:M_r]}^{t-1}, W_a, \mathcal{H}^n)
\end{aligned}$$

where (f) follows from the fact that conditioning reduces entropy.

APPENDIX B

PROOF OF LEMMA 2

We use Lemma 1, as follows:

$$\begin{aligned}
\frac{1}{c_1} h(Y_{a[1:M_r]}^n | W_a, \mathcal{H}^n) &\geq \frac{1}{c_1} h(Y_{a[1:c_1]}^n | W_a, \mathcal{H}^n) \\
&= \frac{1}{c_1} \sum_{t=1}^n h(Y_{a[1:c_1],t} | Y_{a[1:c_1]}^{t-1}, W_a, \mathcal{H}^n) \\
&\stackrel{(g)}{\geq} \frac{1}{c_1} \sum_{t=1}^n h(Y_{a[1:c_1],t} | Y_{a[1:M_r]}^{t-1}, W_a, \mathcal{H}^n) \\
&\stackrel{(h)}{\geq} \frac{1}{c_2} \sum_{t=1}^n h(Y_{a[1:c_1],t}, Y_{b[1:c_2-c_1],t} | Y_{a[1:M_r]}^{t-1}, Y_{b[1:M_r]}^{t-1}, W_a, \mathcal{H}^n)
\end{aligned}$$

$$\begin{aligned}
&= \frac{1}{c_2} \sum_{t=1}^n \left[h(Y_{a[1:M_r],t}, Y_{b[1:M_r],t} | Y_{a[1:M_r],t}^{t-1}, Y_{b[1:M_r],t}^{t-1}, W_a, \mathcal{H}^n) \right. \\
&\quad \left. - h(Y_{a[c_1+1:M_r],t}, Y_{b[c_2-c_1+1:M_r],t} | Y_{a[1:c_1],t}, Y_{b[1:c_2-c_1],t}, Y_{a[1:M_r],t}^{t-1}, Y_{b[1:M_r],t}^{t-1}, W_a, \mathcal{H}^n) \right] \\
&\stackrel{(i)}{\geq} \frac{1}{c_2} h(Y_{a[1:M_r]}^n, Y_{b[1:M_r]}^n | W_a, \mathcal{H}^n) + n \cdot o(\log_2 P)
\end{aligned} \tag{B.1}$$

where $c_1 = \min(M_r, M_t + M_c)$, $c_2 = \min(2M_r, M_t + M_c)$, (g) follows from the fact that conditioning reduces entropy, and (h) follows from Lemma 1, (i) follows from the fact that for all $t \in \{1, \dots, n\}$,

$$\begin{aligned}
&h(Y_{a[c_1+1:M_r],t}, Y_{b[c_2-c_1+1:M_r],t} | Y_{a[1:c_1],t}, Y_{b[1:c_2-c_1],t}, Y_{a[1:M_r],t}^{t-1}, Y_{b[1:M_r],t}^{t-1}, W_a, \mathcal{H}^n) \\
&= h(Y_{a[c_1+1:M_r],t}, Y_{b[c_2-c_1+1:M_r],t} | Y_{a[1:c_1],t}, Y_{b[1:c_2-c_1],t}, Y_{a[1:M_r],t}^{t-1}, Y_{b[1:M_r],t}^{t-1}, X_a^n, W_a, \mathcal{H}^n) \\
&\leq o(\log_2 P)
\end{aligned} \tag{B.2}$$

because $Y_{a[c_1+1:M_r],t}$ and $Y_{b[c_2-c_1+1:M_r],t}$ do not affect the DoF when the channel inputs X_a^n and the channel outputs $Y_{a[1:c_1],t}$ and $Y_{b[1:c_2-c_1],t}$ are given. Therefore, we have

$$\frac{1}{c_1} h(Y_{a[1:M_r]}^n | W_a, \mathcal{H}^n) \geq \frac{1}{c_2} h(Y_{a[1:M_r]}^n, Y_{b[1:M_r]}^n | W_a, \mathcal{H}^n) + n \cdot o(\log_2 P).$$

REFERENCES

- [1] M. Kang and W. Choi, “The capacity of a three-user interference channel with a cognitive transmitter in strong interference,” in *Proc. IEEE Int. Symp. Info. Theory*, Cambridge, MA, Jul. 2012.
- [2] O. Sahin and E. Erkip, “On achievable rates for interference relay channel with interference cancellation,” in *Proc. Asilomar Conf. Signals, Systems and Computers*, Pacific Grove, CA, Nov. 2007.
- [3] M. Costa, “Writing on dirty paper (corresp.),” *IEEE Trans. Info. Theory*, vol. 29, no. 3, pp. 439-441, May 1983.
- [4] S. Sridharan, S. Vishwanath, S. A. Jafar, and S. Shamai, “On the capacity of cognitive relay assisted Gaussian interference channel,” in *Proc. IEEE Int. Symp. Info. Theory*, Toronto, Ontario, Canada, Jul. 2008.
- [5] T. S. Han and K. Kobayashi, “A new achievable rate region for the interference channel,” *IEEE Trans. Info. Theory*, vol. 27, no. 1, pp. 4960, Jan. 1981.
- [6] S. Rini, D. Tuninetti, and N. Devroye, “Outer bounds for the interference channel with a cognitive relay,” in *Proc. IEEE Information Theory Workshop*, Dublin, Ireland, Sep. 2010.
- [7] S. Rini, D. Tuninetti, N. Devroye, and A. Goldsmith, “The capacity of the interference channel with a cognitive relay in strong interference,” in *Proc. IEEE Int. Symp. Info. Theory*, St. Petersburg, Russia, Jul. 2011.

- [8] —, “On the capacity of the interference channel with a cognitive relay,” *IEEE Trans. Info. Theory*, vol. 60, no. 4, pp. 2148-2179, Apr. 2014.
- [9] H. Charnchi, G. A. Hodtani, and M. Nasiri-Kenari, “A new outer bound for a class of interference channels with a cognitive relay and a certain capacity result,” *IEEE Communications Letters*, vol. 17, no. 2, pp. 241-244, Feb. 2013.
- [10] A. Dytso, S. Rini, N. Devroye, and D. Tuninetti, “On the capacity region of the two-user interference channel with a cognitive relay,” *IEEE Trans. Wireless Commun.*, vol. 13, no. 12, pp. 6824-6838, Dec. 2014.
- [11] A. Chaaban and A. Sezgin, “The DoF of the K-user interference channel with a cognitive relay,” in *Proc. IEEE Int. Symp. Info. Theory*, Cambridge, MA, Jul. 2012.
- [12] C. Wang and A. Sezgin, “Degrees of freedom of the interference channel with a cognitive helper,” *IEEE Communications Letters*, vol. 17, no. 5, pp. 920-923, May 2013.
- [13] R. A. Tannious and A. Nosratinia, “Relay-assisted interference network: degrees of freedom,” *IEEE Trans. Info. Theory*, vol. 58, no. 6, pp. 3780-3786, Jun. 2012.
- [14] V. R. Cadambe and S. A. Jafar, “Interference alignment and degrees of freedom of the K-user interference channel,” *IEEE Trans. Info. Theory*, vol. 54, no. 8, pp. 3425-3441, Aug. 2008.
- [15] M. A. Maddah-Ali and D. Tse, “Completely stale transmitter channel state information is still very useful,” *IEEE Trans. Info. Theory*, vol. 58, no. 7, pp. 4418-4431, Jul. 2012.
- [16] C. S. Vaze and M. K. Varanasi, “The degrees of freedom region of the two-user MIMO broadcast channel with delayed CSIT,” in *Proc. IEEE Int. Symp. Info. Theory*, St. Petersburg, Russia, Jul. 2011.
- [17] M. J. Abdoli, A. Ghasemi, and A. K. Khandani, “On the degrees of freedom of three-user MIMO broadcast channel with delayed CSIT,” in *Proc. IEEE Int. Symp. Info. Theory*, St. Petersburg, Russia, Jul. 2011.
- [18] H. Maleki, S. A. Jafar, and S. Shamai, “Retrospective interference alignment over interference networks,” *IEEE J. Sel. Topics in Sig. Processing*, vol. 6, no. 3, pp. 228-240, Jun. 2012.
- [19] M. J. Abdoli, A. Ghasemi, and A. K. Khandani, “On the degrees of freedom of K -user SISO interference and X channels with delayed CSIT,” Available: <http://arxiv.org/abs/1109.4314>, 2011.
- [20] A. Ghasemi, A. S. Motahari, and A. K. Khandani, “On the degrees of freedom of X channel with delayed CSIT,” in *Proc. IEEE Int. Symp. Info. Theory*, St. Petersburg, Russia, Jul. 2011.
- [21] M. G. Kang and W. Choi, “Ergodic interference alignment with delayed feedback,” *IEEE Sig. Processing Letters*, vol. 20, no. 5, pp. 511-514, May 2013.
- [22] C. S. Vaze and M. K. Varanasi, “The degrees of freedom region and interference alignment for the MIMO interference channel with delayed CSIT,” *IEEE Trans. Info. Theory*, vol. 58, no. 7, pp. 4396-4417, Jul. 2012.
- [23] R. Tandon, S. Mohajer, H. V. Poor, and S. Shamai, “Degrees of freedom region of the MIMO interference channel with output feedback and delayed CSIT,” *IEEE Trans. Info. Theory*, vol. 59, no. 3, pp. 1444-1457, Mar. 2013.

- [24] A. Ghasemi, A. S. Motahari, and A. K. Khandani, "Interference alignment for the MIMO interference channel with delayed local CSIT," Available: <http://arxiv.org/abs/1102.5673>, 2011.
- [25] K. Mohanty, C. S. Vaze, and M. K. Varanasi, "The degrees of freedom region for the MIMO interference channel with hybrid CSIT," *IEEE Trans. Info. Theory*, to be published.
- [26] C. Huang, S. A. Jafar, S. Shamai, and S. Vishwanath, "On degrees of freedom region of MIMO networks without channel state information at transmitters," *IEEE Trans. Info. Theory*, vol. 58, no. 2, pp. 849-857, Feb. 2012.
- [27] C. S. Vaze and M. K. Varanasi, "The degree-of-freedom regions of MIMO broadcast, interference, and cognitive radio channels with no CSIT," *IEEE Trans. Info. Theory*, vol. 58, no. 8, pp. 5354-5374, Aug. 2012.
- [28] A. B. Carleial, "Interference channels," *IEEE Trans. Info. Theory*, vol. 24, no. 1, pp. 6070, Jan. 1978.
- [29] N. Devroye, P. Mitran and V. Tarokh, "Achievable rates in cognitive radio channels," *IEEE Trans. Info. Theory*, vol. 52, no. 5, pp. 1813-1827, May. 2006.
- [30] I. Maric, R. D. Yates and G. Kramer, "Capacity of interference channels with partial transmitter cooperation," *IEEE Trans. Info. Theory*, vol. 53, no. 10, pp. 3536-3548, Oct. 2007.
- [31] I. Maric, A. Goldsmith, G. Kramer and S. Shamai, "On the capacity of interference channels with one cooperating transmitter," *Eur. Trans. Telecomms*, vol. 19, no. 4, pp. 405-420, Jun. 2008.
- [32] S. Rini and A. Goldsmith, "On the capacity of the MIMO cognitive interference channel," in *Proc. IEEE Int. Symp. Info. Theory*, Istanbul, Turkey, Jul. 2013.
- [33] —, "On the capacity of the multiantenna Gaussian cognitive interference channel," *IEEE J. Sel. Areas Commun.*, vol. 32, no. 11, pp. 2252-2267, Nov. 2014.
- [34] C. Huang and S. A. Jafar, "Degrees of freedom of the MIMO interference channel with cooperation and cognition," *IEEE Trans. Info. Theory*, vol. 55, no. 9, pp. 4211-4220, Sep. 2009.